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STATISTICS

FORMULAS FOR 4ST601

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Descriptive Statistics

$$p_i = \frac{n_i}{n} \qquad \sum_{i=1}^{k} n_i = n \qquad \sum_{i=1}^{k} p_i = 1 \qquad i = 1, 2, ..., k$$

$$x_p = x_P \qquad n \cdot \frac{P}{100} < z_P < n \cdot \frac{P}{100} + 1 \qquad n\,p < z_p < n\,p + 1$$

$$n\frac{P}{100} + 0.5 = z_P \qquad np + 0.5 = z_p$$

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad \overline{x} = \frac{\sum_{i=1}^{k} x_i\, n_i}{\sum_{i=1}^{k} n_i} \qquad \overline{x} = \sum_{i=1}^{k} x_i\, p_i$$

$$\overline{x}_H = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}} \qquad \overline{x}_H = \frac{\sum_{i=1}^{k} n_i}{\sum_{i=1}^{k} \frac{n_i}{x_i}} \qquad \overline{x}_H = \frac{1}{\sum_{i=1}^{k} \frac{p_i}{x_i}}$$

$$\overline{x}_G = \sqrt[n]{\prod_{i=1}^{n} x_i} = \sqrt[n]{x_1 \cdot x_2 \cdot \ldots \cdot x_n} \qquad \overline{x}_G = \sqrt[n]{\prod_{i=1}^{k} x_i^{n_i}} = \sqrt[n]{x_1^{n_1} \cdot x_2^{n_2} \cdot \ldots \cdot x_k^{n_k}}$$

$R = x_{\max} - x_{\min}$

$$s_x^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n} \qquad s_x^2 = \overline{x^2} - \overline{x}^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} - \left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2$$

$$s_x^2 = \frac{\sum_{i=1}^{k} (x_i - \overline{x})^2 n_i}{\sum_{i=1}^{k} n_i} \qquad s_x^2 = \overline{x^2} - \overline{x}^2 = \frac{\sum_{i=1}^{k} x_i^2\, n_i}{\sum_{i=1}^{k} n_i} - \left(\frac{\sum_{i=1}^{k} x_i\, n_i}{\sum_{i=1}^{k} n_i}\right)^2$$

$$s_x^2 = \sum_{i=1}^{k} (x_i - \overline{x})^2 p_i \qquad s_x^2 = \overline{x^2} - \overline{x}^2 = \sum_{i=1}^{k} x_i^2\, p_i - \left(\sum_{i=1}^{k} x_i\, p_i\right)^2$$

$$s_x^2 = \overline{s^2} + s_{\overline{x}}^2 = \frac{\sum_{i=1}^{k} s_i^2\, n_i}{\sum_{i=1}^{k} n_i} + \frac{\sum_{i=1}^{k} (\overline{x}_i - \overline{x})^2\, n_i}{\sum_{i=1}^{k} n_i} \qquad \overline{x} = \frac{\sum_{i=1}^{k} \overline{x}_i\, n_i}{\sum_{i=1}^{k} n_i}$$

$$s_x^2 = \sum_{i=1}^{k} s_i^2 p_i + \sum_{i=1}^{k} (\overline{x}_i - \overline{x})^2 p_i \qquad \overline{x} = \sum_{i=1}^{k} \overline{x}_i\, p_i$$

$$s_x = \sqrt{s_x^2} \qquad V_x = \frac{s_x}{x}$$

Probability

Probability Theory

$$P(A) = \frac{m}{n}$$

$P(A \cup B) = P(A) + P(B)$ $\quad$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Random Variables

$P(x) = P(X = x)$ $\quad$ $F(x) = P(X \le x) = \sum_{t \le x} P(t)$ $\quad$ $P(x_1 < X \le x_2) = \sum_{x_1 < x \le x_2} P(x) = F(x_2) - F(x_1)$

$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)dt$ $\quad$ $f(x) = F'(x)$ $\quad$ $\int_{-\infty}^{\infty} f(x)dx = 1$

$P(x_1 < X \le x_2) = \int_{x_1}^{x_2} f(x)dx = F(x_2) - F(x_1)$

x_p $\quad$ $F(x_p) = p$

$E(X) = \sum_x x\, P(x)$ $\quad$ $E(X) = \int_{-\infty}^{\infty} x\, f(x)dx$

$\sigma^2 = \sigma^2(X) = \sum_x x^2 P(x) - \left[\sum_x x P(x)\right]^2$ $\quad$ $\sigma^2(X) = \int_{-\infty}^{\infty} x^2 f(x)dx - \left[\int_{-\infty}^{\infty} x f(x)dx\right]^2$

$\sigma = \sigma(X) = \sqrt{\sigma^2(X)}$

Probability Distributions

Bernoulli Distribution (Alternative Distribution) $A[\pi]$

$P(x) = \pi^x (1-\pi)^{1-x}$ $\quad$ $x = 0, 1, \; 0 < \pi < 1$

$E(X) = \pi$ $\quad$ $\sigma^2(X) = \pi(1-\pi)$

Binomial Distribution $Bi[n;\pi]$

$P(x) = \binom{n}{x} \pi^x (1-\pi)^{n-x}$ $\quad$ $x = 0, 1, 2, ..., n, \; n > 0, \; 0 < \pi < 1$

$E(X) = n\pi$ $\quad$ $\sigma^2(X) = n\pi(1-\pi)$

Poisson Distribution $Po[\lambda]$

$P(x) = e^{-\lambda} \frac{\lambda^x}{x!}$ $\quad$ $x = 0, 1, ... , \lambda > 0,$ $\quad$ $E(X) = \lambda$ $\quad$ $\sigma^2(X) = \lambda$

Hypergeometric Distribution $Hy[N;M;n]$

$$P(x)=\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}},\qquad x = max(0, M\text{-}N\text{+}n), ..., min(M, n),\ n>0,\ N\geq n,\ M\leq N$$

$$E(X)=n\frac{M}{N}\qquad \sigma^2(X)=n\frac{M}{N}\left(1-\frac{M}{N}\right)\frac{N-n}{N-1}$$

Normal Distribution $N[\mu;\sigma^2]$

$$-\infty<x<\infty,\ -\infty<\mu<\infty,\ \sigma^2>0\qquad E\left(X\right)=\mu\qquad \sigma^2(X)\ =\ \sigma^2$$

$$u=\frac{x-\mu}{\sigma}\qquad F(x)=\Phi(u)=\Phi\left(\frac{x-\mu}{\sigma}\right)\qquad x_p=\mu+\sigma u_p$$

$$P(x_1\leq X\leq x_2)=P\left(\frac{x_1-\mu}{\sigma}\leq\frac{X-\mu}{\sigma}\leq\frac{x_2-\mu}{\sigma}\right)=\ P(u_1\leq U\leq u_2)=\Phi(u_2)-\Phi(u_1)$$

Standard Normal Distribution $N[0;1]$

$$U=\frac{X-\mu}{\sigma}\qquad E\left(U\right)=0\qquad \sigma^2(U)\ =\ 1$$

$$\Phi(u)=1-\Phi(\text{-}u)\qquad \Phi(\text{-}u)=1-\Phi(u)\qquad u_p=-u_{1-p}$$

Chi-Square Distribution $\chi^2[\nu]$ $\quad x>0$

t-Distribution (Student's Distribution) $t[\nu]$

$$-\infty\ <\ x\ <\ \infty\qquad t_p\left[\nu\right]\ =\ -\ t_{1-p}\left[\nu\right]$$

F-Distribution (*Fisher's – Snedecor's D.*) $F[\nu_1;\nu_2]$ $\quad x>0\quad F_p[\nu_1;\nu_2]=\dfrac{1}{F_{1-p}[\nu_2;\nu_1]}$

Elements of Sampling and Statistical Inference

$$s'_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

Parameter's Estimates

Population Mean (Expected Value) est $\mu = \hat{\mu} = \bar{x}$ est $N\mu = N\,\bar{x}$

Normal distribution

a) σ^2 known

$$P\left(\bar{x} - u_{1-\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + u_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

$$P\left(\bar{x} - u_{1-\alpha}\frac{\sigma}{\sqrt{n}} < \mu\right) = 1-\alpha \qquad P\left(\mu < \bar{x} + u_{1-\alpha}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

b) σ^2 unknown

$$P\left(\bar{x} - t_{1-\alpha/2}\frac{s'_x}{\sqrt{n}} < \mu < \bar{x} + t_{1-\alpha/2}\frac{s'_x}{\sqrt{n}}\right) = 1-\alpha \qquad t \sim t[n-1]$$

$$P\left(\bar{x} - t_{1-\alpha}\frac{s'_x}{\sqrt{n}} < \mu\right) = 1-\alpha \qquad P\left(\mu < \bar{x} + t_{1-\alpha}\frac{s'_x}{\sqrt{n}}\right) = 1-\alpha$$

General distribution, $\sigma^2(X)$ unknown, large sample ($n > 30$)

$$P\left(\bar{x} - u_{1-\alpha/2}\frac{s'_x}{\sqrt{n}} < E(X) < \bar{x} + u_{1-\alpha/2}\frac{s'_x}{\sqrt{n}}\right) = 1-\alpha$$

$$P\left(\bar{x} - u_{1-\alpha}\frac{s'_x}{\sqrt{n}} < E(X)\right) = 1-\alpha \qquad P\left(E(X) < \bar{x} + u_{1-\alpha}\frac{s'_x}{\sqrt{n}}\right) = 1-\alpha$$

Variance σ^2 (Normal Distribution) est $\sigma^2 = \hat{\sigma}^2 = s_x^{2\prime}$

Population Proportion (parameter π of Bernoulli Distribution)

est $\pi = \hat{\pi} = p$ est $N\pi = Np$

$$P\left(p - u_{1-\alpha/2}\sqrt{\frac{p(1-p)}{n}} < \pi < p + u_{1-\alpha/2}\sqrt{\frac{p(1-p)}{n}}\right) = 1-\alpha$$

$$P\left(p - u_{1-\alpha}\sqrt{\frac{p(1-p)}{n}} < \pi\right) = 1-\alpha \qquad P\left(\pi < p + u_{1-\alpha}\sqrt{\frac{p(1-p)}{n}}\right) = 1-\alpha$$

Hypotheses Testing

Population Mean, Normal Distribution

H_0	H_1	Test Statistic (Test Criterion)	Rejection Region
$\mu = \mu_0$	$\mu > \mu_0$ $\mu < \mu_0$ $\mu \neq \mu_0$	σ^2 known $U = \dfrac{\bar{x} - \mu_0}{\sigma}\sqrt{n}$ $\quad U \sim N[0;1]$	$W_\alpha = \{U \geq u_{1-\alpha}\}$ $W_\alpha = \{U \leq -u_{1-\alpha}\}$ $W_\alpha = \{\lvert U\rvert \geq u_{1-\alpha/2}\}$
		σ^2 unknown $t = \dfrac{\bar{x} - \mu_0}{s'_x}\sqrt{n}$ $\quad t \sim t[n-1]$	$W_\alpha = \{t \geq t_{1-\alpha}\}$ $W_\alpha = \{t \leq -t_{1-\alpha}\}$ $W_\alpha = \{\lvert t\rvert \geq t_{1-\alpha/2}\}$

Expected Value, General Distribution, Large Sample

H_0	H_1	Test Statistic (Test Criterion)	Rejection Region
$E(X) = \mu_0$	$E(X) > \mu_0$ $E(X) < \mu_0$ $E(X) \neq \mu_0$	σ^2 unknown ($n > 30$) $U = \dfrac{\bar{x} - \mu_0}{s'_x}\sqrt{n}$ $\quad U \approx N[0;1]$	$W_\alpha = \{U \geq u_{1-\alpha}\}$ $W_\alpha = \{U \leq -u_{1-\alpha}\}$ $W_\alpha = \{\lvert U\rvert \geq u_{1-\alpha/2}\}$

Population Proportion (Large Samples)

H_0	H_1	Test Statistic (Test Criterion)	Rejection Region
$\pi = \pi_0$	$\pi > \pi_0$ $\pi < \pi_0$ $\pi \neq \pi_0$	$U = \dfrac{p - \pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}}$ $\quad U \approx N[0;1]$	$W_\alpha = \{U \geq u_{1-\alpha}\}$ $W_\alpha = \{U \leq -u_{1-\alpha}\}$ $W_\alpha = \{\lvert U\rvert \geq u_{1-\alpha/2}\}$

Equality of Two Population's Means

Large independent samples

H_0	H_1	Test Statistic (Test Criterion)	Rejection Region
$\mu_1 = \mu_2$ $\mu_1 - \mu_2 = 0$	$\mu_1 > \mu_2$ $\mu_1 < \mu_2$ $\mu_1 \neq \mu_2$	σ_1^2, σ_2^2 unknown $U = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^{2\prime}}{n_1} + \dfrac{s_2^{2\prime}}{n_2}}}$ $\quad U \approx N[0;1]$	$W_\alpha = \{U \geq u_{1-\alpha}\}$ $W_\alpha = \{U \leq -u_{1-\alpha}\}$ $W_\alpha = \{\lvert U\rvert \geq u_{1-\alpha/2}\}$

Dependent samples from normal distribution (paired *t*-test)

H_0	H_1	Test Statistic (Test Criterion)	Rejection Region
$\mu_1 = \mu_2$ $\mu_1 - \mu_2 = 0$	$\mu_1 > \mu_2$ $\mu_1 < \mu_2$ $\mu_1 \neq \mu_2$	$t = \dfrac{\bar{d}}{s'_d}\sqrt{n}$ $\quad t \sim t[n-1]$ $d_i = x_{1i} - x_{2i}$, $i = 1,2,..,n$	$W_\alpha = \{t \geq t_{1-\alpha}\}$ $W_\alpha = \{t \leq -t_{1-\alpha}\}$ $W_\alpha = \{\lvert t\rvert \geq t_{1-\alpha/2}\}$

Chi-Square Goodnes-of-Fit Test

H_0 and H_1	Test Statistic (Test Criterion)	Rejection Region
H_0: $\pi_i = \pi_{0,i}$ $\quad i = 1, .., k$ H_1: non H_0	$G = \sum_{i=1}^{k} \dfrac{(n_i - n\pi_{0,i})^2}{n\pi_{0,i}}$ $\quad G \approx \chi^2[k\text{-}1]$	$W_\alpha = \{G \geq \chi^2_{1-\alpha}\}$ $n\pi_{0,i} \geq 5$

Analysis of Dependence

Contingency Table (*r* x c)

$$n_{i.}=\sum_{j=1}^{c}n_{ij}\qquad n_{.j}=\sum_{i=1}^{r}n_{ij}\qquad n'_{ij}=\frac{n_{i.}n_{.j}}{n}\qquad n'_{ij}\geq 5$$

H_0	H_1	Test Statistic (Test Criterion)	Rejection Region
$\pi_{ij}=\pi_{i.}\,\pi_{.j}$ $1\leq i\leq r$ $1\leq j\leq c$	non H_0	$G=\sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(n_{ij}-n'_{ij})^2}{n'_{ij}}$ $\quad G\approx\chi^2[(r-1)(c-1)]$	$W_\alpha=\{G\geq\chi^2_{1-\alpha}\}$

$$C=\sqrt{\frac{G}{n+G}}\qquad V=\sqrt{\frac{G}{n(m-1)}},\ m=\min(r,c)$$

Table 2 x 2 $\qquad G=n\dfrac{(n_{11}n_{22}-n_{12}n_{21})^2}{n_{1.}n_{2.}n_{.1}n_{.2}}$

Analysis of Variance

$$S_T=\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(x_{ij}-\overline{x}\right)^2=S_{tr}+S_E\qquad S_{tr}=\sum_{i=1}^{k}\left(\overline{x}_i-\overline{x}\right)^2 n_i\qquad S_E=\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(x_{ij}-\overline{x}_i\right)^2$$

$$P^2=\frac{S_{tr}}{S_T}\qquad P=\sqrt{P^2}$$

H_0	H_1	Test Statistic (Test Criterion)	Rejection Region
$\mu_1=\mu_2=\ldots=\mu_k$	non H_0	$F=\dfrac{\frac{S_{tr}}{k-1}}{\frac{S_E}{n-k}}$ $\quad F\sim F[k-1;n-k]$	$W_\alpha=\{F\geq F_{1-\alpha}\}$

Regression and Correlation

Regresssion line $y=\beta_0+\beta_1 x+\varepsilon,\quad Y_i=b_0+b_1 x_i\quad$ minimum$_{b_0,b_1}\ \sum_{i=1}^{n}(y_i-b_0-b_1x_i)^2$

$$s_{xy}=\frac{\sum_{i=1}^{n}\left(x_i-\overline{x}\right)\left(y_i-\overline{y}\right)}{n}=\overline{xy}-\overline{x}\cdot\overline{y}$$

$$b_1=b_{yx}=\frac{n\sum y_ix_i-\sum x_i\sum y_i}{n\sum x_i^2-\left(\sum x_i\right)^2}=\frac{\overline{xy}-\overline{x}\cdot\overline{y}}{\overline{x^2}-\overline{x}^2}=\frac{s_{xy}}{s_x^2}$$

$$b_0=\frac{\sum y_i\sum x_i^2-\sum y_ix_i\sum x_i}{n\sum x_i^2-(\sum x_i)^2}=\overline{y}-b_{yx}\overline{x}$$

Other regression functions $Y_i = b_0 + b_1 x_i + b_2 x_i^2$ $Y_i = b_0 + b_1 x_{1i} + b_2 x_{2i} + .. + b_k x_{ki}$

$$S_T = \sum_{i=1}^{n} (y_i - \bar{y})^2 \qquad S_R = \sum_{i=1}^{n} (Y_i - \bar{y})^2$$

$$s_y^2 = \frac{1}{n}\sum_{i=1}^{n} (y_i - \bar{y})^2 = \frac{S_T}{n} \qquad s_R^2 = \frac{1}{n}\sum_{i=1}^{n} (Y_i - \bar{y})^2 = \frac{S_R}{n}$$

$$S_E = \sum_{i=1}^{n} (y_i - Y_i)^2 = \sum_{i=1}^{n} e_i^2 \qquad s_{(T-R)}^2 = \frac{1}{n}\sum_{i=1}^{n} (y_i - Y_i)^2 = \frac{S_E}{n} \qquad s_E^2 = \frac{S_E}{n-p}$$

$$S_T = S_R + S_E \qquad s_y^2 = s_R^2 + s_{(T-R)}^2 \qquad s_E = \sqrt{\frac{S_E}{n-p}} = \sqrt{s_E^2}$$

$$R_{yx}^2 = R^2 = \frac{S_R}{S_T} \qquad R_{yx} = \sqrt{R_{yx}^2} \qquad R_{\text{ADJ}}^2 = 1 - (1 - R_{yx}^2)\frac{n-1}{n-p}$$

Hypothesis Test of Regression Parameters

H_0	H_1	Test Statistic (Test Criterion)	Rejection Region
$\beta_i = 0$	$\beta_i \neq 0$	$t = \dfrac{b_i}{s(b_i)}$ $\quad t \sim t[n-p]$	$W_\alpha = \{\lvert t \rvert \geq t_{1-\alpha/2}\}$

Hypothesis Test of Model $p = k + 1$

H_0	H_1	Test Statistic (Test Criterion)	Rejection Region
$\beta_0 = c$ $\beta_1 = 0$ ... $\beta_k = 0$	non H_0	$F = \dfrac{\frac{S_R}{p-1}}{\frac{S_E}{n-p}}$ $\quad F \sim F[p-1; n-p]$	$W_\alpha = \{F \geq F_{1-\alpha}\}$

Correlation Coefficient

$$r_{yx} = r_{xy} = r = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}\sqrt{n\sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} y_i)^2}} = \frac{\overline{xy} - \bar{x}\cdot\bar{y}}{\sqrt{\left(\overline{x^2} - \bar{x}^2\right)\left(\overline{y^2} - \bar{y}^2\right)}} = \frac{s_{xy}}{s_x s_y}$$

H_0	H_1	Test Statistic (Test Criterion)	Rejection Region
$\rho_{yx} = 0$	$\rho_{yx} \neq 0$	$t = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ $\quad t \sim t[n-2]$	$W_\alpha = \{\lvert t \rvert \geq t_{1-\alpha/2}\}$

Time Series

$$\bar{y} = \frac{\sum_{t=1}^{n} y_t}{n} \qquad \bar{y} = \frac{\frac{1}{2} y_1 + \sum_{t=2}^{n-1} y_t + \frac{1}{2} y_n}{n-1} \qquad \bar{y} = \frac{\frac{y_1 + y_2}{2} d_1 + \frac{y_2 + y_3}{2} d_2 + \ ... \ + \frac{y_{n-1} + y_n}{2} d_{n-1}}{d_1 + d_2 + \ ... \ + d_{n-1}}$$

$$\Delta_t = y_t - y_{t-1} \qquad \bar{\Delta} = \frac{1}{n-1} \sum_{t=2}^{n} \Delta_t = \frac{y_n - y_1}{n-1}$$

$$k_t = \frac{y_t}{y_{t-1}} \qquad \bar{k} = \sqrt[n-1]{k_2 k_3 ... k_n} = \sqrt[n-1]{\frac{y_n}{y_1}}$$

$$I_{t/1} = \frac{y_t}{y_1} = I_{2/1} \ I_{3/2} ... I_{t/t-1} \qquad I_{t/t-1} = \frac{y_t}{y_{t-1}} = \frac{I_{t/1}}{I_{t-1/1}}$$

Moving Averages

$m = 2p + 1$

$$\bar{y}_t = \frac{\sum_{i=-p}^{p} y_{t+i}}{m} = \frac{y_{t-p} + ... + y_{t-1} + y_t + y_{t+1} + ... + y_{t+p}}{m}$$

$m = 2p$

$$\bar{y}_t = \frac{1}{2m}\left(y_{t-p} + 2y_{t-p+1} + . + 2y_{t-1} + 2y_t + 2y_{t+1} + . + 2y_{t+p-1} + y_{t+p} \right)$$

Time Series Decomposition

$y_t = T_t + S_t + C_t + \varepsilon_t \qquad y_t = T_t S_t C_t \varepsilon_t$

$T_t = \beta_0 + \beta_1 t \qquad \hat{T}_t = b_0 + b_1 t$

$T_t = \beta_0 + \beta_1 t + \beta_2 t^2 \qquad \hat{T}_t = b_0 + b_1 t + b_2 t^2$

$$\text{MSE} = \frac{\sum_{t=1}^{n} (y_t - \hat{T}_t)^2}{n}$$

Regression Method with Dummy Variables (linear trend, length of seasonality 4)

$y_t = T_t + S_t + \varepsilon_t = \beta_0 + \beta_1 t + \alpha_1 x_{1t} + \alpha_2 x_{2t} + \alpha_3 x_{3t} + \varepsilon_t$

$$\bar{a} = \frac{a_1 + a_2 + a_3}{4} \qquad S_{i+4j} = a_i - \bar{a} \quad i = 1,2,3 \quad S_{4+4j} = -\bar{a} \qquad \hat{T}_t = (b_0 + \bar{a}) + b_1 t$$

Index Numbers Analysis

$Q = pq$

$$Ip = \frac{p_1}{p_0} \qquad \Delta p = p_1 - p_0 \qquad Iq = \frac{q_1}{q_0} \qquad \Delta q = q_1 - q_0 \qquad IQ = \frac{Q_1}{Q_0} \qquad \Delta Q = Q_1 - Q_0$$

$$I(\Sigma q) = \frac{\sum q_1}{\sum q_0} = \frac{\sum Iq \cdot q_0}{\sum q_0} = \frac{\sum q_1}{\sum \frac{q_1}{Iq}} \qquad \Delta(\Sigma q) = \sum q_1 - \sum q_0$$

$$I(\Sigma Q) = \frac{\sum Q_1}{\sum Q_0} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = \frac{\sum IQ \cdot Q_0}{\sum Q_0} = \frac{\sum Q_1}{\sum \frac{Q_1}{IQ}} \qquad \Delta(\Sigma Q) = \sum Q_1 - \sum Q_0$$

$$I\overline{p} = \frac{\overline{p}_1}{\overline{p}_0} = \frac{\frac{\sum Q_1}{\sum q_1}}{\frac{\sum Q_0}{\sum q_0}} = \frac{\frac{\sum p_1 q_1}{\sum q_1}}{\frac{\sum p_0 q_0}{\sum q_0}} = \frac{\frac{\sum Q_1}{\sum \frac{Q_1}{p_1}}}{\frac{\sum Q_0}{\sum \frac{Q_0}{p_0}}} \qquad \Delta\overline{p} = \overline{p}_1 - \overline{p}_0 = \frac{\sum p_1 q_1}{\sum q_1} - \frac{\sum p_0 q_0}{\sum q_0}$$

$$Ip^{(L)} = \frac{\sum p_1 q_0}{\sum p_0 q_0} = \frac{\sum Ip \cdot p_0 q_0}{\sum p_0 q_0} = \frac{\sum Ip \cdot Q_0}{\sum Q_0} \qquad Ip^{(P)} = \frac{\sum p_1 q_1}{\sum p_0 q_1} = \frac{\sum p_1 q_1}{\sum \frac{p_1 q_1}{Ip}} = \frac{\sum Q_1}{\sum \frac{Q_1}{Ip}}$$

$$Ip^{(F)} = \sqrt{Ip^{(L)} \; Ip^{(P)}}$$

$$Iq^{(L)} = \frac{\sum p_0 q_1}{\sum p_0 q_0} = \frac{\sum Iq \cdot p_0 q_0}{\sum p_0 q_0} = \frac{\sum Iq \cdot Q_0}{\sum Q_0} \qquad Iq^{(P)} = \frac{\sum p_1 q_1}{\sum p_1 q_0} = \frac{\sum p_1 q_1}{\sum \frac{p_1 q_1}{Iq}} = \frac{\sum Q_1}{\sum \frac{Q_1}{Iq}}$$

$$Iq^{(F)} = \sqrt{Iq^{(L)} \; Iq^{(P)}}$$