Basic examples and calculations in life insurance

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Example 1
A client, man 25 years old, wants to make an insurance contract for 25 years.

A. In case of death he wants to secure his family with 200 000. In case he survives all 25 years he also wants to receive 200 000. How much would this contract cost him?

Derive results of single and regular netto premium.

B. The client wants to see how much of the premium he pays for insurance to secure his family in case of death and how much for 200 000 in case he survives.

Derive results of single and regular premium.

C. You, as an insurance company, should be able to cover most of your contracts. For that purpose you should calculate reserves for all considered contracts. Calculate reserve in year 7.

D. Apply charges and calculate brutto premium and reserves of contracts mentioned in section A and B.

*note: assume traditional approach with interest rate 0.025 p.a.

A. Endowment

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>25</td>
</tr>
<tr>
<td>Policy period</td>
<td>25</td>
</tr>
<tr>
<td>Death benefit (K)</td>
<td>200 000</td>
</tr>
<tr>
<td>Survival benefit (D)</td>
<td>200 000</td>
</tr>
<tr>
<td>Interest rate (i)</td>
<td>0.025</td>
</tr>
<tr>
<td>Type of insurance contract</td>
<td>Endowment</td>
</tr>
<tr>
<td>Premium:</td>
<td>Endowment</td>
</tr>
<tr>
<td>Single</td>
<td>108 888.99</td>
</tr>
<tr>
<td>Regular annual</td>
<td>5 829.87</td>
</tr>
<tr>
<td>Regular monthly</td>
<td>491.46</td>
</tr>
</tbody>
</table>

Single premium
Using Excel function

Using Mortality tables

\[
\Pi_{x:n} = K \cdot \frac{d_x \cdot v + d_{x+1} \cdot v^2 + \ldots + d_{x+n-1} \cdot v^n}{l_x} + D \cdot \frac{l_{x+n}}{l_x} \cdot v^n
\]

| 108 888.99 = 200 000 \cdot 2626.83 + 200 000 \cdot 95039.86 | \cdot 0.53939 |
| 98982.34 | 98982.34 |
Using probability

\[ \Pi_{xn} = K \times q_{x \cdot v + 1} q_{x \cdot v} + \ldots + n \cdot q_{x \cdot v^n} + D \times p_{x \cdot v^n} \]

\[ 108\,888,99 = 200\,000 \times 0,02654 + 200\,000 \times 0,96017 \times 0,539391 \]

Using Commutation numbers

\[ \Pi_{xn} = K \times \frac{C_x + C_{x+1} + C_{x+2} + \ldots + C_{x+n-1}}{D_x} + D \times \frac{D_{x+n}}{D_x} \]

\[ 108\,888,99 = 200\,000 \times \frac{1416,89}{53390,15} + 200\,000 \times \frac{27651,11}{53390,15} \]

Regular premium – annual

Using Excel function

\[ f_x = \text{regular\_Endowment}(25; 25; 0,025; 200000; 200000) \times \text{200000} \]

Using Actuarial formulas

\[ K \times p_{x} = \frac{K \times A_{\alpha_{xn}}^{x} + D \times nE_{x}}{\alpha_{xn}^{x}} \]

\[ 5\,829,87 = 200\,000 \times 0,02654 + 200\,000 \times 0,51791 \]

Using probability

\[ K \times p_{x} = \frac{K \times q_{x \cdot v + 1} q_{x \cdot v} + \ldots + n \cdot q_{x \cdot v^n} + D \times p_{x \cdot v^n}}{1 + 1p_{x \cdot v} + \ldots + n-1p_{x \cdot v^{n-1}}} \]

\[ 5\,829,87 = 200\,000 \times 0,02654 + 200\,000 \times 0,51791 \]

\[ 18,68 \]
Using Commutation numbers

\[
K \times P_x = \frac{K \times (M_x - M_{x+n}) + D \times (D_{x+n})}{N_x - N_{x+n}}
\]

<table>
<thead>
<tr>
<th>5 829,87</th>
<th>200 000</th>
<th>*</th>
<th>1416,89</th>
<th>+</th>
<th>200 000</th>
<th>*</th>
<th>27651,11</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>997208,20</td>
</tr>
</tbody>
</table>

\[
K \times P_x = \frac{K \times d_x \cdot v + d_{x+1} \cdot v^2 + \ldots + d_{x+n} \cdot v^n + D \times l_{x+n} \cdot v^n}{l_x + l_{x+1} \cdot v + l_{x+2} \cdot v^2 + \ldots + l_{x+n} \cdot v^n}
\]

<table>
<thead>
<tr>
<th>5 829,87</th>
<th>200 000</th>
<th>*</th>
<th>2626,83</th>
<th>+</th>
<th>200 000</th>
<th>*</th>
<th>51263,61</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1848768,26</td>
</tr>
</tbody>
</table>

Regular premium – monthly

<table>
<thead>
<tr>
<th>Frequency of premium (m)</th>
<th>monthly</th>
<th>m = 12</th>
</tr>
</thead>
</table>
| \( K \times P_x^{(m)} \) = | \( \frac{\text{Regular netto premium}}{m \times (1 - \frac{(m-1) \times (D_x - D_{x+n})}{2m \times (N_x - N_{x+n})})} \)
| 491,46 | = | 5 829,87 |
| | | 11,86 |
B. Pure endowment and Term insurance

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>25</td>
</tr>
<tr>
<td>Policy period</td>
<td>25</td>
</tr>
<tr>
<td>Death benefit (K)</td>
<td>200 000</td>
</tr>
<tr>
<td>Survival benefit (D)</td>
<td>200 000</td>
</tr>
<tr>
<td>Interest rate (i)</td>
<td>0.025</td>
</tr>
<tr>
<td>Type of insurance contract</td>
<td>Endowment / Term insurance</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Premium:</th>
<th>Endowment</th>
<th>Pure endowment</th>
<th>Term insurance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>108 888,99</td>
<td>103 581,31</td>
<td>5 307,67</td>
</tr>
<tr>
<td>Regular annual</td>
<td>5 828,87</td>
<td>5 545,70</td>
<td>284,17</td>
</tr>
<tr>
<td>Regular monthly</td>
<td>491,46</td>
<td>467,51</td>
<td>23,96</td>
</tr>
</tbody>
</table>

Pure endowment – single premium

*Using Excel function*

\[
\text{Using Mortality tables}
\]

\[
\Pi_{x+n} = D \times \frac{l_{x+n}}{l_x} \times v^n
\]

\[
103 581,31 = 200 000 \times \frac{95039,86}{95390,15} \times 0,53939
\]

*Using probability*

\[
\Pi_{x+n} = D \times n_p x \times v^n
\]

\[
103 581,31 = 200 000 \times \frac{0,96017}{0,53939}
\]

*Using Commutation numbers*

\[
\Pi_{x+n} = D \times \frac{D_{x+n}}{D_x}
\]

\[
103 581,31 = \frac{27651,11}{53390,15}
\]
Pure endowment – Regular premium – annually

Using Excel function

\[ \text{fs} = \text{regular Pure Endowment}(25;25;0,025) \times 200000 \]

Using Actuarial formulas

\[ K \times P_x = D \times \frac{nE_x}{\overline{a}_{x+1}} \]

\[
\begin{align*}
5545,70 & = 200000 \times 0,51791 \\
& = 18,68
\end{align*}
\]

Using Commutation numbers

\[ K \times P_x = D \times \frac{D_{x+n}}{N_x - N_{x+n}} \]

\[
\begin{align*}
5545,70 & = 200000 \times 27651,11 \\
& = 997208,20
\end{align*}
\]

Using Mortality tables

\[ K \times P_x = D \times \frac{l_{x+n} \times v^n}{l_x + l_{x+1} \times v + l_{x+2} \times v^2 + \ldots + l_{x+n-1} \times v^{n-1}} \]

\[
\begin{align*}
5545,70 & = 200000 \times 51263,61 \\
& = 1848768,26
\end{align*}
\]

Using probabilities

\[ K \times P_x = D \times \frac{nP_x \times V^n}{1 + 2P_x \times V + \ldots + n-1P_x \times V^{n-1}} \]

\[
\begin{align*}
5545,70 & = 200000 \times 0,51791 \\
& = 18,68
\end{align*}
\]
### Pure endowment – Regular premium – monthly

<table>
<thead>
<tr>
<th>Frequency of premium (m)</th>
<th>monthly</th>
<th>( m = 12 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ K \times nP_x^{(m)} = \frac{\text{Regular netto premium}}{m \times (1 - \frac{(m-1) \cdot (D_x - D_{x+n})}{2m \cdot (N_x - N_{x+n})})} ]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ 467,51 = \frac{5\,545,70}{11,86} \]

### Term insurance – Single premium

Using Excel function:

\[ f_x = A1\times n(25;25;0,025)^*200000 \]

Using Mortality tables

\[ \Pi_{xn} = K \times d_x + V + d_{x+1} \cdot V^2 + \ldots + d_{x+n-1} \cdot V^n \]

\[ 5\,307,67 = 200\,000 \times 2626,83 \]

Using probabilities

\[ \Pi_{xn} = K \times q_x \cdot V + q_{x+1} \cdot V^2 + \ldots + q_{x+n-1} \cdot V^n \]

\[ 5\,307,67 = 200\,000 \times 0,02654 \]

Using Commutation numbers

\[ \Pi_{xn} = K \times \frac{C_x + C_{x+1} + C_{x+2} + \ldots + C_{x+n-1}}{D_x} = \frac{M_x - M_{x+n}}{D_x} \]

\[ 5\,307,67 = 200\,000 \times \frac{1416,89}{53390,15} = 1416,89 \]
Term insurance – Regular premium – annually

Using Excel function

\[ \text{Using Actuarial formulas} \]
\[
\begin{align*}
\text{Using Commutation numbers} \\
\text{Using Mortality tables} \\
\text{Using probabilities} \\
\end{align*}
\]

Term insurance – Regular premium - monthly

<table>
<thead>
<tr>
<th>Frequency of premium (m)</th>
<th>Anually</th>
<th>m = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K \times nP_x^{(m)} )</td>
<td>( \text{Regular netto premium} )</td>
<td></td>
</tr>
<tr>
<td>( m \times (1 - \frac{(m-1) \times (D_x - D_{x+n})}{2m \times (N_x - N_{x+n})}) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23,96</td>
<td>284,17</td>
<td></td>
</tr>
<tr>
<td>11,86</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
C. Reserves

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td>Male</td>
</tr>
<tr>
<td>Age</td>
<td>25</td>
</tr>
<tr>
<td>Policy period</td>
<td>25</td>
</tr>
<tr>
<td>Death benefit (K)</td>
<td>200 000</td>
</tr>
<tr>
<td>Survival benefit (D)</td>
<td>200 000</td>
</tr>
<tr>
<td>Interest rate (i)</td>
<td>0.025</td>
</tr>
<tr>
<td>Reserve year (t)</td>
<td>7</td>
</tr>
<tr>
<td>Type of insurance contract</td>
<td>Endowment / Term insurance / Pure endowment</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Premium:</th>
<th>Reserve of regular premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endowment</td>
<td>$0.22055 \times 200 000 = 44 110$</td>
</tr>
<tr>
<td>Pure endowment</td>
<td>$0.21517 \times 200 000 = 43 034$</td>
</tr>
<tr>
<td>Term insurance</td>
<td>$0.00540 \times 200 000 = 1 080$</td>
</tr>
</tbody>
</table>

**Endowment**

Using Excel function

\[ f_x = tV_x \text{ Endowment}(25;25;7;0,025;1;200000;200000) \times 200000 \]

**Using Actuarial formulas**

\[ iV_x = A_{x+t,n-t} - \deltaP_x \times \delta_{x+t,n-t} \]

\[
0.22055 = 0.64492 - 0.02915 \times 14.56
\]

**Using Commutation numbers**

\[ iV_x = 1 - \frac{D_x}{D_{x+t}} \times \frac{N_{x+t} - N_{x+n}}{N_x - N_{x+n}} \]

\[
0.22055 = 1 - \frac{53390.15}{44680.94} \times \frac{650482.57}{997208.20}
\]

**Netto reserves**
Pure endowment
Using Excel function

\[ f_x = tV_x_{\text{Pure endowment}}(25;25;7;0,025;1) \times 2000000 \]

Using Actuarial formulas

\[ tV_x = n-tE_{xt} - nP_x \times \dot{a}_{xt,n-t} \]

\[
\begin{array}{cccc}
0.21517 & = & 0.61886 - & 0.02773 \times 14.56 \\
\end{array}
\]

Using Commutation numbers

\[ tV_x = \frac{D_{x+n}}{D_{xt}} \times \frac{N_x - N_{xt}}{N_x - N_{x+n}} \]

\[
\begin{array}{cccc}
0.21517 & = & \frac{27651,11}{44680,94} \times \frac{346725,63}{997208,20} \\
\end{array}
\]

Netto reserves
**Term insurance**

Using Excel function:

\[ \text{Sum} = \sum\text{Vx}_{\text{Term insurance}}(25;25;7;0.025;1)*2000000 \]

**Using Actuarial formulas**

\[ V_x = A_{x+t,n}^1 - nP_x \times \bar{a}_{x+t,n} \]

\[ V_x = 0.0054 = 0.0261 - 0.0014 \times 14,5584 \]

**Using Commutation numbers**

\[ V_x = \frac{M_{x+t} - M_{x+n}}{D_{x+t}} - \frac{M_x - M_{x+n}}{D_{x+t}} \times \frac{N_{x+t} - N_{x+n}}{N_x - N_{x+n}} \]

\[ V_x = 0.0054 = \frac{1164,4039}{44680,9390} - \frac{1416,8875}{44680,9390} \times \frac{650482,5696}{997208,2031} \]

---

**D. Brutto premium and reserves**

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>25</td>
</tr>
<tr>
<td>Policy period</td>
<td>25</td>
</tr>
<tr>
<td>Death benefit (K)</td>
<td>200 000</td>
</tr>
<tr>
<td>Survival benefit (D)</td>
<td>200 000</td>
</tr>
<tr>
<td>Interest rate (i)</td>
<td>0.025</td>
</tr>
<tr>
<td>Reserve year (t)</td>
<td>7</td>
</tr>
<tr>
<td>Type of insurance contract</td>
<td>Endowment / Term insurance / Pure endowment</td>
</tr>
<tr>
<td>Premium</td>
<td>Regular brutto</td>
</tr>
<tr>
<td>------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Endowment</td>
<td>5 965,93</td>
</tr>
<tr>
<td>Pure endowment</td>
<td>5 680,87</td>
</tr>
<tr>
<td>Term insurance</td>
<td>402,92</td>
</tr>
</tbody>
</table>

*Note: There are no Excel functions to calculate brutto premium

**Endowment**

**Endowment brutto premium**

\[ B_{xn} = \frac{K \cdot A_{xn} + \alpha^x + \alpha_{fix} + \bar{\alpha}_{xn} \cdot (\beta_{fix} + \beta^x + \gamma_{fix} + \gamma^x)}{\bar{\alpha}_{xn} \cdot (1 - \beta^x_{brutto} - \gamma^x_{brutto}) - \alpha_{brutto}} \]

<table>
<thead>
<tr>
<th>5 965,93</th>
<th>108888,99</th>
<th>2004,00</th>
<th>18,68</th>
<th>10,20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>18,68</td>
<td>0,99690</td>
<td>-</td>
<td>0,0002</td>
</tr>
</tbody>
</table>

**Endowment brutto reserve**

\[ K^* V^*_{x_{brutto}} = K^* V^*_{x_{netto}} - \frac{\left(\alpha^x + \alpha_{fix} + \alpha_{brutto}\right) \cdot \bar{\alpha}_{x+t, n}}{\bar{\alpha}_{x,n}} \]

<table>
<thead>
<tr>
<th>42 546,92</th>
<th>44109,87</th>
<th>2005,19</th>
<th>14,56</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>18,68</td>
<td></td>
</tr>
</tbody>
</table>

**Brutto reserves**

- \( K^* V^*_{x_{netto}} \)
- \( K^* V^*_{x_{brutto}} \)
**Pure endowment**

**Pure endowment brutto premium**

\[
K \cdot B_{xn}^{brutto} = D \cdot E_x + \alpha^k + \alpha^{fix} + \delta_{xn} \cdot (\beta^{fix} + \beta^k + \gamma^{fix} + \gamma^k)
\]

\[
= 5 680,87 + 204,00 + 18,68 \cdot 10,20
\]

**Pure endowment netto reserve**

\[
K^* V_x^{brutto} = K^* V_x^{netto} - \frac{(\alpha^k + \alpha^{fix} + \alpha^{Bxn}) \cdot \delta_{x+t,n-t}}{\delta_{x,n}}
\]

\[
= 41 471,95 - \frac{2005,14 \cdot 14,56}{18,68}
\]
**Term insurance**

**Term insurance brutto premium**

\[
B_{xn} = K \cdot A_{xn}^1 + \alpha^K + \alpha^{\text{fix}} + \ddot{a}_{xn} \cdot (\beta^{\text{fix}} + \beta^K + \gamma^{\text{fix}} + \gamma^K)
\]

\[
\ddot{a}_{xn} \cdot (1 - \beta^{\text{fix}} - \gamma^{\text{fix}}) - \alpha^B_{xn}
\]

\[
\begin{align*}
402.92 &= 5307.67 + 2004.00 + 18.68 \cdot 10.20 \\
\end{align*}
\]

**Term insurance brutto reserve**

\[
V_{x}^{\text{brutto}} = V_{x}^{\text{netto}} - \frac{(\alpha^K + \alpha^{\text{fix}} + \alpha^B_{xn})}{\ddot{a}_{x,n}} \cdot \ddot{a}_{x+1,n+1}
\]

\[
\begin{align*}
-487.07 &= 1075.01 - \frac{2004.08}{18.68} \\
\end{align*}
\]

---

**Brutto reserves**

![Brutto reserves graph](image)
Example 2
50 years old female wants to be insured for one million in case of death.

A. Compare single and regular premium for whole life.
B. Calculate also reserve at age 75.
C. Calculate whole life premium and reserve including charges.

A. Whole life

<table>
<thead>
<tr>
<th>Gender</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>50</td>
</tr>
<tr>
<td>Policy period</td>
<td>75</td>
</tr>
<tr>
<td>Death benefit (K)</td>
<td>1 000 000</td>
</tr>
<tr>
<td>Type of insurance contract</td>
<td>Whole life</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Premium:</th>
<th>Single premium</th>
<th>Regular premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole life</td>
<td>453 687,44</td>
<td>20 254,98</td>
</tr>
</tbody>
</table>

Whole life – single premium
Using Excel function:

\[ \Pi_x = A1x(50;0,025) \times 1000000 \]

Using Mortality tables

\[ \Pi_x = K \times \frac{d_x \cdot v + d_{x+1} \cdot v^2 + \ldots}{l_x} \]

\[ 453 687,44 = 1 000 000 \times 44291,15 \]
\[ = 97624,80 \]

Using probability

\[ \Pi_x = K \times q_x \cdot v + q_{x+1} \cdot v^2 + \ldots \]

\[ 453 687,44 = 1 000 000 \times 0,45369 \]

Using commutation numbers

\[ \Pi_x = K \times \frac{C_x + C_{x+1} + \ldots}{D_x} = M_x \]

\[ 453 687,44 = 1 000 000 \times \frac{12886,16}{28403,17} = 12886,16 \]
\[ = 28403,17 \]
**Whole life – Regular premium**

Using Excel function:

\[ f(x) = \text{regular\_Whole\_life}(50;0,025)*1000000 \]

**Using Actuarial formulas**

\[
K * nP_x = K * A_x \\
20,254.98 = 1,000,000 * 0.45369 = 22,398.81
\]

**Using Mortality tables**

\[
K * nP_x = K * d_x * v + d_{x+1} * v^2 + ... + l_{x+n-1} * v^n \\
20,254.98 = 1,000,000 * 4429.15 = 218,667.92
\]

**Using probabilities**

\[
K * nP_x = K * q_x * v + 1 | q_x * v^2 + ... + n | p_x * v^n \\
20,254.98 = 1,000,000 * 0.45 = 22,40
\]

**Using Commutation numbers**

\[
K * nP_x = K * M_x = C_x + C_{x+1} + C_{x+2} + ... \\
20,254.98 = 1,000,000 * 12886.16 = 63,619.45
\]
B. Reserves of Whole life

<table>
<thead>
<tr>
<th>Gender</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>50</td>
</tr>
<tr>
<td>Policy period</td>
<td>( \infty )</td>
</tr>
<tr>
<td>Death benefit (K)</td>
<td>1 000 000</td>
</tr>
<tr>
<td>Survival benefit (D)</td>
<td>0</td>
</tr>
<tr>
<td>Reserve (t)</td>
<td>25</td>
</tr>
<tr>
<td>Type of insurance contract</td>
<td>Whole life</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Premium:</th>
<th>Reserve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole life</td>
<td>0,53529*1M = 535 287,41</td>
</tr>
</tbody>
</table>

Whole life – Reserve

Using Excel function:

\[
\bar{f}_x = tV_x_{\text{Whole life}}(50;25;0,025;1)*1000000
\]

Using Actuarial formulas

\[
tV_x = A^1_{x+t} - P_x * \ddot{a}_{x+t}
\]

\[
0,53529 = 0,74612 - 0,02025 * 10,40901
\]

Using Commutation numbers

\[
tV_x = 1 - \frac{D_x}{D_{x+t}} * \frac{N_{x+t}}{N_x}
\]

\[
0,53529 = 1 - \frac{28403,17}{12285,73} * \frac{127882,26}{636197,45}
\]

Netto reserves
C. Brutto Whole life

<table>
<thead>
<tr>
<th>Gender</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>50</td>
</tr>
<tr>
<td>Policy period</td>
<td>( \omega )</td>
</tr>
<tr>
<td>Death benefit (K)</td>
<td>1 000 000</td>
</tr>
<tr>
<td>Reserve (t)</td>
<td>25</td>
</tr>
<tr>
<td>Type of insurance contract</td>
<td>Whole life</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Premium:</th>
<th>Regular brutto premium</th>
<th>Brutto reserve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole life</td>
<td>20 443,72</td>
<td>534 346,79</td>
</tr>
</tbody>
</table>

Whole life – Regular brutto premium

\[
B_{xn} = \frac{K \times A_x^t + \alpha^K + \alpha^{fix} + \bar{a}_x \times (\beta^{fix} + \beta^K + \gamma^{fix} + \gamma^K)}{\bar{a}_x \times (1 - \beta_{Bxn} - \gamma_{Bxn}) - \alpha_{Bxn}}
\]

\[
20 443,72 = \frac{453687,44 + 2020,00 + 22,40 \times 35,00}{22,40 \times 0,99690 - 0,00020}
\]

Whole life – Brutto reserve

\[
K \times V_{x\text{brutto}} = K \times V_{x\text{netto}} - \frac{(\alpha^K + \alpha^{fix} + \alpha_{Bxn})}{\bar{a}_x} \times \bar{a}_{ext}
\]

\[
534 346,79 = 535287,41 - \frac{2024,09 \times 10,41 \times 22,40}{22,40}
\]
Example 3
Young man of age 30 wants to be secured when he reaches his 60 by certain amount of money.

A. How much he has to pay every year to get two million when he turns 60.
B. How much he would have to pay to get 5 000 every year from 60 until the rest of his life. 
   Compare this premium with premium for 5 000 every year from his 60 until his 80.
C. He is sure that now he can pay max 2000 per year. What annuity he can expect when he turns 60 until his death and until his 80?
D. Make the reserves of these two contracts at year 12 and 45.

A. Pure endowment

Using Actuarial formulas

\[ nP_x = D \times \frac{nE_x}{\ddot{a}_{x+n}} \]

\[ 40\ 680,47 = 2\ 000\ 000 \times \frac{0,42621}{20,95} \]

B. Deferred annuity

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (x)</td>
<td>30</td>
</tr>
<tr>
<td>Policy period (N)</td>
<td>50/(\omega)</td>
</tr>
<tr>
<td>Deferred (k)</td>
<td>30</td>
</tr>
<tr>
<td>Annuity (D)</td>
<td>5 000</td>
</tr>
<tr>
<td>Interest rate (i)</td>
<td>0,025</td>
</tr>
<tr>
<td>Type of insurance contract</td>
<td>Deferred / Whole life / Temporary annuity</td>
</tr>
<tr>
<td>Premium:</td>
<td>Single premium</td>
</tr>
<tr>
<td>Whole life annuity</td>
<td>30 576,34</td>
</tr>
<tr>
<td>Temporary annuity</td>
<td>26 639,88</td>
</tr>
</tbody>
</table>

Whole life annuity – Single premium

Using Excel function

\[ f_x = dax(30;0,025;1;30)*5000 \]

Using Mortality tables

\[ \Pi_x = D \times \frac{l_{x+k+1} \cdot v^{k+1} + l_{x+k+2} \cdot v^{k+2} + \ldots}{l_x} \]

\[ 30\ 576,34 = 5\ 000 \times \frac{603057,53}{98615,07} \]
**Using probabilities**

\[
\Pi_x = \text{D} \ast \sum_{k=1}^{\infty} p_x \cdot v^k + \text{D}_x^* \cdot \sum_{k=1}^{\infty} p_x \cdot v^k + \ldots
\]

| 30 576,34 | 5 000 | * | 6,12 |

**Using Commutation numbers**

\[
\Pi_x = \text{D} \ast \sum_{k=1}^{\infty} D_{x+k+1} + D_{x+k+2} \ldots = \text{N}_{x+k+1}
\]

\[
\text{D}_x \ast \sum_{k=1}^{\infty} D_{x+k+1} = \text{D}_x \ast \text{N}_{x+k+1}
\]

| 30 576,34 | 5 000 | * | 2766364,06 | \text{=} | 287503,27 |
| 47014,01 | 47014,01 |

**Whole life – Regular premium**

Using Excel function

\[
f_x = \text{regular Whole life annuity}(30;0,025;30;1) \ast 5000
\]

<table>
<thead>
<tr>
<th>Regular premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>(nP_x) = (D \ast \frac{k+1}{\ddot{a}_x})</td>
</tr>
<tr>
<td>1 459,22</td>
</tr>
<tr>
<td>20,95</td>
</tr>
</tbody>
</table>

**Temporary annuity – Single premium**

Using Excel function

\[
f_x = \text{Daxn}(30;50;0,025;1;1;30) \ast 5000
\]

**Using Mortality tables**

\[
\Pi_{xn} = \text{D} \ast \sum_{k=1}^{\infty} l_{x+k+1} \cdot v^k + \ldots
\]

| 26 639,88 | 5 000 | * | 525418,84 |
| 98615,07 |
Using probabilities

\[ \Pi_{xn} = D \cdot \left( k+1p_x \cdot v^{k+1} + k+2p_x \cdot v^{k+2} + \ldots + n p_x \cdot v^n \right) \]

\[
\begin{array}{ccc}
26\,639,88 & = & 5\,000 \quad * \\
& & 5,33
\end{array}
\]

Using Commutation numbers

\[ \Pi_{xn} = D \cdot \left( D_{x+k+1} + D_{x+k+2} + \ldots + D_{x+n} \right) = N_{x+k+1} - N_{x+n+1} \]

\[
\begin{array}{ccc}
26\,639,88 & = & 5\,000 \quad * \\
& & 250\,489,59 \quad 47\,014,01
\end{array}
\]

Temporary annuity – Regular premium
Using Excel function

\[ f_x = \text{regular\_Temporary\_annuity}(30;50;0,025;30;1) \times 5000 \]

Regular premium

\[ nP_x = \frac{D \cdot a_{xn}}{\ddot{a}_{nk}} \]

\[
\begin{array}{ccc}
1\,271,36 & = & 26639,88 \quad 20,95
\end{array}
\]

C. Fixed premium annuity

Whole life

\[ P \cdot \ddot{a}_{sk} = D \cdot k|a_s \]

\[ D = \frac{P \cdot \ddot{a}_{sk}}{k|a_s} \]

\[ D = \frac{2000 \cdot 20,95}{6,11} = 685295 \]

Temporary annuity

\[ P \cdot \ddot{a}_{sk} = D \cdot k|a_{sn} \]
\[ D = \frac{P \cdot \ddot{a}_{sk}}{k \cdot a_{aw}} \]

\[ D = \frac{2000 \cdot 20.95}{5.32} = 7865.58 \]

**Excel function for \( \ddot{a}_{sk} \)**

\[
\text{fx} = \text{Daxn}(30;30;0.025;0;1;0)
\]

**Excel function for \( k \cdot a_x \)**

\[
\text{fx} = \text{dax}(30;0.025;1;1;30)
\]

**Excel function for \( k \cdot a_{aw} \)**

\[
\text{fx} = \text{Daxn}(30;30;0.025;0;1;0)
\]
D. Reserves

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (x)</td>
<td>30</td>
</tr>
<tr>
<td>Policy period (N)</td>
<td>50/60</td>
</tr>
<tr>
<td>Deferred (k)</td>
<td>30</td>
</tr>
<tr>
<td>Annuity (D)</td>
<td>5 000</td>
</tr>
<tr>
<td>Interest rate (i)</td>
<td>0,025</td>
</tr>
</tbody>
</table>

Type of insurance contract: Deferred / Whole life / Temporary annuity

<table>
<thead>
<tr>
<th>Premium:</th>
<th>Reserve t = 12</th>
<th>Reserve t = 45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole life annuity</td>
<td>4,16*5 000 = 20 800</td>
<td>7,93*5 000 = 39 650</td>
</tr>
<tr>
<td>Temporary annuity</td>
<td>3,63*5 000 = 18 150</td>
<td>3,98*5 000 = 19 900</td>
</tr>
</tbody>
</table>

Whole life annuity

Using Excel function:

\[ f(x) = \sum V_{x,t} \text{Whole Life annuity}(30;45;0,025;1;30) \times 5000 \]

Using actuarial symbols:

\[ tV_x = k-tP_x \times a_{x+t,k-t} \]

\[ 4,16 = 8,34 \times 0,29 \times 14,30 \]
\[ K=45, \, t \leq k \]

\[
{\text{\( tV_x \)}} = a_{k+t}
\]

\[
7.93 = 7.93
\]

**Temporary annuity**

**Netto reserves**

Using Excel function:

\[
f_{x} = tV_x_{\text{Temporary annuity}}(30;50;12;0,025;1;30)*5000
\]

Using actuarial symbols:

\[ K=12, \, t \text{ for } k > t \]

\[
{\text{\( tV_x \)}} = k-t|a_{k+t,n-t} - \frac{k}{n}P_x \ast a_{k+t,k-t}
\]

\[
3.63 = 7.26 - 0.25 \ast 14.30
\]
$k = 45$, $\nu_x$ for $k \leq t$

<table>
<thead>
<tr>
<th>$\nu_x$</th>
<th>$= a_{x+t,n-t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,98</td>
<td>3,98</td>
</tr>
</tbody>
</table>
Example 4 – Universal Traditional approach

30 years old man has very special demands about his insurance contract. When he turns 40, he wants to be insured for 10 000 in case of death. When he turns 50, he wants to increase death benefit up to 20 000 and until his 55 he wants to receive 5 000 every year. In his 60 he wants to get annuity 20 000 and then until his 70 to be insured for 10 000 in case of death but doesn’t want to pay premium.

The charges for this contract assume following:

<table>
<thead>
<tr>
<th>alfa fix</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>alfa from premium (%)</td>
<td>2,0000%</td>
</tr>
<tr>
<td>alfa z K (%)</td>
<td>2,0000%</td>
</tr>
<tr>
<td>alfa z D (%)</td>
<td>2,0000%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>fix</th>
<th>from K or D</th>
<th>from P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>20,00</td>
<td>2,0000%</td>
</tr>
<tr>
<td>Gamma</td>
<td>30,00</td>
<td>3,0000%</td>
</tr>
<tr>
<td>Delta</td>
<td>35,00</td>
<td>3,5000%</td>
</tr>
</tbody>
</table>

Calculate netto and brutto premium and reserve in policy year 5 and 25 of this contract.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (x)</td>
<td>30</td>
</tr>
<tr>
<td>Policy period (N)</td>
<td>40</td>
</tr>
<tr>
<td>Deferred (k)</td>
<td>10</td>
</tr>
<tr>
<td>Reserve (t)</td>
<td>5 / 25</td>
</tr>
<tr>
<td>Death benefit (K)</td>
<td>From age 40 to 50: 10 000</td>
</tr>
<tr>
<td></td>
<td>From age 50 to 60: 20 000</td>
</tr>
<tr>
<td></td>
<td>From age 60 to 70: 10 000 without paying premium</td>
</tr>
<tr>
<td>Annuity (D)</td>
<td>From age 50 to 55: 5 000</td>
</tr>
<tr>
<td></td>
<td>In age 60: 20 000</td>
</tr>
<tr>
<td>Interest rate (i)</td>
<td>0,025</td>
</tr>
<tr>
<td>Type of insurance contract</td>
<td>Deferred / Term insurance / Temporary annuity</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Premium:</th>
<th>Netto</th>
<th>Brutto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium</td>
<td>1 222,46</td>
<td>2 035,55</td>
</tr>
<tr>
<td>Reserve year 5</td>
<td>6 593,45</td>
<td>5 079,80</td>
</tr>
<tr>
<td>Reserve year 25</td>
<td>16 921,97</td>
<td>16 538,53</td>
</tr>
</tbody>
</table>
The input information of this policy can be seen on picture below.
Example 5 – Flexible product
Deal flexible contract for 35 years old male for 30 years.

A. What is the capital value at the end of policy if the premium is 2 000 per year, death benefit is 100 000 and initial deposit of 1 000?
B. What is the minimal premium to keep zero reserve at the end of policy?
C. What is the premium for similar contract to endowment where the death benefit is 100 000 and survival benefit is 50 000?

A. Capital value at the end of policy

<table>
<thead>
<tr>
<th>Claims:</th>
<th>Policy characteristics</th>
<th>Model point:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Death</td>
<td></td>
<td>Year</td>
</tr>
<tr>
<td>Tech. Int. rate</td>
<td>2,5%</td>
<td>Age at valuation date</td>
</tr>
<tr>
<td>Policy charges:</td>
<td></td>
<td>Sex</td>
</tr>
<tr>
<td>1st year</td>
<td>80,0%</td>
<td>SA</td>
</tr>
<tr>
<td>2+years</td>
<td>20,0%</td>
<td>Premium (annual)</td>
</tr>
<tr>
<td>Profit share</td>
<td>85,0%</td>
<td>CV at val. date</td>
</tr>
<tr>
<td>Surrender fee</td>
<td>5,0%</td>
<td>Policy year at val. date</td>
</tr>
</tbody>
</table>

B. Minimizing premium

<table>
<thead>
<tr>
<th>Year</th>
<th>Age</th>
<th>Policy Year</th>
<th>Pshare</th>
<th>CV EoY</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>64</td>
<td>30</td>
<td>...</td>
<td>67 821</td>
</tr>
</tbody>
</table>

1 *Note: This Solution requires to install solver.
### C. Endowment

#### Policy characteristics

<table>
<thead>
<tr>
<th>Claims:</th>
<th>Death</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tech. Int. rate</td>
<td>2,5%</td>
<td></td>
</tr>
<tr>
<td>Policy charges:</td>
<td>1st year 80,0%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2+years 20,0%</td>
<td></td>
</tr>
<tr>
<td>Profit share</td>
<td>85,0%</td>
<td></td>
</tr>
<tr>
<td>Surrender fee</td>
<td>5,0%</td>
<td></td>
</tr>
</tbody>
</table>

#### Modelpoint:

<table>
<thead>
<tr>
<th>Year</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age at valuation date</td>
<td>35</td>
</tr>
<tr>
<td>Sex</td>
<td>Male</td>
</tr>
<tr>
<td>SA</td>
<td>100 000</td>
</tr>
<tr>
<td>Premium (annual)</td>
<td>1 616</td>
</tr>
<tr>
<td>CV at val. date</td>
<td>1 000</td>
</tr>
<tr>
<td>Policy year at val. date</td>
<td>1</td>
</tr>
<tr>
<td>Policy_period</td>
<td>30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Age</th>
<th>Policy Year</th>
<th>Pshare</th>
<th>CV EoY</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>2044</td>
<td>64</td>
<td>30</td>
<td>...</td>
<td>50 000</td>
</tr>
</tbody>
</table>
Example 6 – Cash-Flow model
One big company wants to insure 1 000 its employees. In case of death, 100 000 will be paid and if insured person survives 10 years the contract will be canceled and the person will obtain at least 20 000. All employees are all men in age 35.

A. Compare traditional and flexible approach and decide which of the two types of contract is more profitable.
B. Make calculation of Liability Adequacy Test (LAT) for both approaches.

A. Profitability test

Flexible approach
Calculation of flexible product premium with SA = 100 000 and at the end CV = 20 000. By using solver the minimal premium is **2 495**.

From Cash-Flow model the profitable criteria is **Pcrit 1**.

\[
P_{crit 1} = \frac{PVPL}{PV \text{ Premium}}
\]

<table>
<thead>
<tr>
<th>Policy characteristics</th>
<th>Modelpoint:</th>
<th>Year 2015</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Claims:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Death</td>
<td>SA</td>
<td></td>
</tr>
<tr>
<td>Tech. Int. Rate</td>
<td>2,5%</td>
<td></td>
</tr>
<tr>
<td><strong>Policy charges:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st year</td>
<td>80,0%</td>
<td></td>
</tr>
<tr>
<td>2+years</td>
<td>20,0%</td>
<td></td>
</tr>
<tr>
<td>Profit share</td>
<td>85,0%</td>
<td></td>
</tr>
<tr>
<td>Surrender fee</td>
<td>5,0%</td>
<td></td>
</tr>
</tbody>
</table>

| Premium (annual) | 2 495 |

| Age at valuation date | 35 |
| Sex                  | M   |
| SA                   | 100 000 |
| CV at val. date      | 0   |
| Policy year at val. date | 1   |
| Policy period       | 10  |

<table>
<thead>
<tr>
<th>Liability model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PV CF</td>
<td>-29 388 850</td>
</tr>
<tr>
<td>PV PL</td>
<td>-29 636 250</td>
</tr>
<tr>
<td>PV Premium</td>
<td>115 266 345</td>
</tr>
<tr>
<td>Net PL</td>
<td>-24 830 982</td>
</tr>
<tr>
<td>Total Earnings</td>
<td>-24 830 982</td>
</tr>
<tr>
<td>Pcrit 1</td>
<td>-25,7%</td>
</tr>
<tr>
<td>Pcrit 2</td>
<td>-118,8%</td>
</tr>
</tbody>
</table>
**Traditional approach**

The premium of traditional approach of endowment contract below needs to be firstly calculated.

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>Age (x)</th>
<th>In force</th>
<th>Premium transfer</th>
<th>Death benefit</th>
<th>Survival benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
<td>1</td>
<td>1</td>
<td>100 000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>36</td>
<td>1</td>
<td>1</td>
<td>100 000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>37</td>
<td>1</td>
<td>1</td>
<td>100 000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>38</td>
<td>1</td>
<td>1</td>
<td>100 000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>39</td>
<td>1</td>
<td>1</td>
<td>100 000</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>40</td>
<td>1</td>
<td>1</td>
<td>100 000</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>41</td>
<td>1</td>
<td>1</td>
<td>100 000</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>42</td>
<td>1</td>
<td>1</td>
<td>100 000</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>43</td>
<td>1</td>
<td>1</td>
<td>100 000</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>44</td>
<td>1</td>
<td>1</td>
<td>100 000</td>
<td>20 000</td>
</tr>
</tbody>
</table>

**Regular brutto premium**

2 146.58

The profit criteria can be seen in output of Cash-flow model.

**Liability model**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PV CF</td>
<td>-14 196 104</td>
</tr>
<tr>
<td>PV PL</td>
<td>5 468 347</td>
</tr>
<tr>
<td>PV Premium</td>
<td>99 171 010</td>
</tr>
<tr>
<td>Net PL</td>
<td>4 106 974</td>
</tr>
<tr>
<td>Total Earnings</td>
<td>4 907 146</td>
</tr>
<tr>
<td>Pcrit 1</td>
<td>5.5%</td>
</tr>
<tr>
<td>Pcrit 2</td>
<td>25.5%</td>
</tr>
</tbody>
</table>

Traditional approach seems to be more profitable for insurance company based on Pric1 and also for insured person because of lower premium. The reason why traditional approach gives better results is based on assumptions of no surrenders payoffs. If client cancels flexible policy, he receives his capital value adjusted by surrender fee. In traditional approach we assume no payout when the policy is canceled.
B. Liability Adequacy Test

<table>
<thead>
<tr>
<th>Flexible approach</th>
<th>Traditional approach</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BE</strong></td>
<td><strong>BE</strong></td>
</tr>
<tr>
<td>29 388 850</td>
<td>14 196 104</td>
</tr>
<tr>
<td><strong>RM</strong></td>
<td><strong>RM</strong></td>
</tr>
<tr>
<td>8 976 977</td>
<td>10 882 614</td>
</tr>
<tr>
<td><strong>FV</strong></td>
<td><strong>FV</strong></td>
</tr>
<tr>
<td>38 365 827</td>
<td>25 078 718</td>
</tr>
<tr>
<td><strong>LAT</strong></td>
<td><strong>LAT</strong></td>
</tr>
<tr>
<td>38 365 827</td>
<td>5 074 425</td>
</tr>
</tbody>
</table>

After pressing “Calculate LAT” Liability adequacy test will be automatically calculated.
Actuarial formulas and MS Excel functions

Nomenclature

$X$  
Age  

$N$  
Policy period  

$i$  
Interest rate  

$D$  
Death benefit  

$K$  
Survival benefit  

$t$  
Time of reserve  

$k$  
Deferred time

Mortality tables and Commutation tables

**Probability of death $q_x$**
Excel function: $qx(x)$

$$q_x = \frac{l_x - l_{x+1}}{l_x} = \frac{d_x}{l_x}$$

**Probability of survive $p_x$**
Excel function: $px(x)$

$$p_x = 1 - q_x$$

$$p_x = \frac{l_{x+1}}{l_x}$$

**Number of living $l_x$**
Excel function: $lx(x)$

$$l_x = p_{x-1} \cdot l_{x-1} = (1 - q_{x-1}) \cdot l_{x-1}$$

**Number of death $d_x$**
Excel function: $dx(x)$

$$d_x = l_x - l_{x+1}$$

**Probability of surviving $n$ years $np_x$**
Excel function: $npx(x,n)$

$$np_x = \frac{l_{x+n}}{l_x}$$
\[ n \, p_x = \prod_{i=0}^{n} p_{x+i} \]

**Probability of death in n years \( n q_x \)**
Excel function: \( n q_x (x,n) \)

\[ n \, q_x = \frac{l_x - l_{x+n}}{l_x} \]

\[ n \, q_x = 1 - n \, p_x \]

**Probability of death in certain age \( x+n \) \( n q_x \)**
Excel function: \( n \_q x (x,n) \)

\[ n \, q_x = \frac{d_{x+n}}{l_x} \]

**Discounted number of living at age \( x \)**
Excel function: \( D D x (x,i) \)

\[ D_x = l_x \cdot v^x \]

**Discounted number of death at age \( x \)**
Excel function: \( C x (x,i) \)

\[ C_x = d_x \cdot v^{x+1} \]

**Commutation numbers of first order**
Excel function: \( M x (x,i) \)

\[ N_x = D_x^{[2]} = \sum_{i=0}^{x+n} D_{x+i} \]

Excel function: \( N x (x,i) \)

\[ M_x = C_x^{[2]} = \sum_{i=0}^{x+n} C_{x+i} \]

**Commutation numbers of second order**
Excel function: \( S x (x,i) \)

\[ S_x = D_x^{[3]} = \sum_{i=0}^{x+n} N_{x+i} \]

Excel function: \( R x (x,i) \)

\[ R_x = C_x^{[3]} = \sum_{i=0}^{x+n} M_{x+i} \]
Actuarial functions

**Pure endowment**
Excel function: \( nEx(x,n,i) \)

\[
\begin{align*}
\text{\( a_E_x = \frac{l^{x+n} \cdot v^n}{l_x} \)} \\
\text{\( nE_x = \frac{D_x^{x+n}}{D_x} \)} \\
\text{\( nE_x = p_x \cdot v^n \)}
\end{align*}
\]

**Whole life**
Excel function: \( A1x(x,i) \)

\[
\begin{align*}
\text{\( A_1^1 = \sum_{i=0}^\infty \frac{d_x^{x+i} \cdot v^{i+1}}{l_x} \)} \\
\text{\( A_1^x = \frac{\sum_{i=0}^\infty C_x^{x+i}}{D_x} = \frac{M_x}{D_x} \)} \\
\text{\( A_1^x = \sum_{i=0}^\infty q_x^{x+i} \cdot v^{i+1} \)}
\end{align*}
\]

**Temp insurance**
Excel function: \( A1xn(x,n,i) \)

\[
\begin{align*}
\text{\( A_{1n}^1 = \sum_{i=0}^{n-1} \frac{d_x^{x+i} \cdot v^{i+1}}{l_x} \)} \\
\text{\( A_{1n}^x = \frac{\sum_{i=0}^{n-1} C_x^{x+i}}{D_x} = \frac{M_x - M_{x+n}}{D_x} \)} \\
\text{\( A_{1n}^x = \sum_{i=0}^{n-1} q_x^{x+i} \cdot v^{i+1} \)}
\end{align*}
\]

**Endowment**
Excel function: \( Axn(x,n,I,D,K) \)

\[
\begin{align*}
\text{\( A_{sn} = A_{1n}^1 + D / K \cdot n \cdot E_x \)} \\
\text{\( A_{sn} = \sum_{i=0}^{n-1} \frac{d_x^{x+i} \cdot v^{i+1}}{l_x} + D / K \cdot \frac{l_x^{x+n} \cdot v^n}{l_x} \)}
\end{align*}
\]
\[
A_{x\mid n} = \sum_{i=0}^{n-1} \frac{C_{x+i}}{D_x} + D / K \cdot \frac{D_{x+n}}{D_x} = M_x - M_{x+n} + D / K \cdot \frac{D_{x+n}}{D_x}
\]

\[
A_{x\mid n} = \sum_{i=0}^{n-1} \ln q_{x+i} \cdot v^{i+1} + D / K \cdot p_x \cdot v^n
\]

**Whole life annuity**

Excel function: \(Dax(x,i,\text{in}_\text{arearrs},\text{frequency},\text{deferred})\)

\[
k\mid a_x = \sum_{i=0}^{\infty} l_{x+k+i} \cdot v^{k+i}
\]

\[
k\mid a_x = \sum_{i=0}^{\infty} k+i p_x \cdot v^{k+i}
\]

\[
k\mid a_x = \sum_{i=0}^{\infty} D_{x+k+i} \cdot N_{x+k+1} = D_x
\]

\[
a_x = \ddot{a}_x - 1
\]

\[
k\mid \ddot{a}_x = \sum_{i=0}^{\infty} l_{x+k+i} \cdot v^{k+i}
\]

\[
k\mid \ddot{a}_x = \sum_{i=0}^{\infty} k+i p_x \cdot v^{k+i}
\]

\[
k\mid a_x = \frac{\sum_{i=0}^{\infty} D_{x+k+i}}{D_x} = N_{x+k} = N_{x+k+1}
\]

**Temp annuity**

Excel function: \(Daxn(x,N,i,\text{in}_\text{arearrs},\text{frequency},\text{deferred})\)

\[
k\mid a_{x\mid n} = \sum_{i=0}^{n} l_{x+k+i} \cdot v^{k+i}
\]

\[
k\mid a_{x\mid n} = \sum_{i=0}^{n} k+i p_x \cdot v^{k+i}
\]

\[
k\mid a_{x\mid n} = \sum_{i=0}^{n} D_{x+k+i} \cdot N_{x+k+n+1} = D_x
\]

\[
a_{x\mid n} = \ddot{a}_{x\mid n} - 1
\]

\[
k\mid \ddot{a}_{x\mid n} = k\cdot a_{x\mid n}
\]
\[ k \ddot{a}_{xn} = \sum_{i=0}^{n-1} l_{x+i} \cdot v^{k+i} \]

\[ k \ddot{a}_{xn} = \sum_{i=0}^{n-1} P_{x+i} \cdot v^{k+i} \]

\[ k \ddot{a}_{xn} = \sum_{i=0}^{n-1} \frac{D_{x+i}}{D_x} = \frac{N_{x+k+n}}{D_x} \]

**Reserves**

**Endowment reserve**
Excel function: \( tVx\_Endowment (x, n, t) \)

\[ tV_x = A_{x+t, n-t} - P_{2n} \cdot \ddot{a}_{x+t, n-t} \]

\[ tV_x = 1 - \frac{D_x}{D_{x+t}} \cdot \frac{N_{x+t} - N_{x+n}}{N_x - N_{x+n}} \]

**Whole life reserve**
Excel function: \( tVx\_Whole\_live (x, t) \)

\[ tV_x = A^1_{x+t} - P_{2n} \cdot \ddot{a}_{x+t} \]

\[ tV_x = 1 - \frac{D_x}{D_{x+t}} \cdot \frac{N_{x+t}}{N_x} \]

**Temp insurance reserve**
Excel function: \( tVx\_Temp\_insurance (x,n,t) \)

\[ tV_x = A^1_{x+t, n-t} - P_{2n} \cdot \ddot{a}_{x+t, n-t} \]

\[ tV_x = \frac{M_{x+t} - M_{x+n} - M_x - M_{x+n}}{D_{x+t}} \cdot \frac{N_{x+t} - N_{x+n}}{N_x - N_{x+n}} \]

**Pure endowment reserves**
Excel function: \( tVx\_Pure\_endowment (x,n,t) \)

\[ tV_x = E_{x+t} - P_{2n} \cdot \ddot{a}_{x+t, n-t} \]

\[ tV_x = \frac{D_{x+n}}{D_{x+t}} \cdot \frac{N_x - N_{x+t}}{N_x - N_{x+n}} \]

**Deferred life anuity reserves**
Excel function:
For $t<k$

$$V_x = k - \ddot{a}_{x+t} - k \bar{a}_{x+k-t}$$

$$V_x = \frac{\bar{N}_{x+k}}{D_{x+t}} \frac{N_x - N_{x+k}}{N_x - N_{x+k}}$$

For $t\geq k$

$$V_x = \ddot{a}_{x+t}$$

$$iV_x = \frac{N_{x+t}}{D_{x+t}}$$

**Regular netto premium**

**Pure endowment regular**

Excel function: \textit{regular\_Pure\_endowment} $(x,n,i)$

$$P_{an} = \frac{n E_x}{\ddot{a}_{an}}$$

$$P_{an} = \frac{D_{x+n}}{N_x - N_{x+n}}$$

$$P_{an} = \frac{l_{x+n} \cdot v^n}{\sum_{i=0}^{n-1} l_{x+i} \cdot v^i}$$

$$P_{an} = \frac{n p_x \cdot v^n}{\sum_{i=0}^{n-1} p_x \cdot v^i}$$

**Whole life regular**

Excel function: \textit{regular\_Whole\_life} $(x,n,i)$

$$P_x = \frac{A_x}{\ddot{a}_x}$$

$$P_x = \frac{M_x}{N_x} \frac{\sum_{i=0}^{n} C_{x+i}}{N_x}$$

$$P_x = \frac{\sum_{i=0}^{n} d_{x+i} \cdot v^{i+1}}{\sum_{i=0}^{n} l_{x+i} \cdot v^i}$$

$$P_x = \frac{\sum_{i=0}^{n} d_{x+i} \cdot v^{i+1}}{\sum_{i=0}^{n} l_{x+i} \cdot v^i}$$
\[ P_x = \frac{\sum_{i=0}^{n} q_x \cdot v^{i+1}}{\sum_{i=0}^{n} p_x \cdot v^{i}} \]

**Temp insurance regular**

Excel function: `regular_netto_Temp_insurance(x,n,i)`

\[ P_{x+n} = \frac{A_{x+n}^1}{\bar{a}_{x+n}} + \frac{M - M_{x+n}}{N_{x+n}} = \frac{\sum_{i=0}^{n-1} C_{x+i}}{N_{x+n}} - N_{x+n} \]

\[ P_{x+n} = \frac{\sum_{i=0}^{n-1} d_{x+i} \cdot v^{i+1}}{\sum_{i=0}^{n} l_{x+i} \cdot v^{i}} \]

\[ P_x = \frac{\sum_{i=0}^{n-1} q_x \cdot v^{i+1}}{\sum_{i=0}^{n} p_x \cdot v^{i}} \]

**Endowment regular**

Excel function: `regular_netto_Endowment(x,n,i,K,D)`

\[ P_{x+n} = \frac{A_{x+n}^1 \cdot D \cdot K \cdot E_x}{\bar{a}_{x+n}} \]

\[ P_{x+n} = \frac{M - M_{x+n} \cdot D \cdot D_{x+n}}{N_{x+n} - N_{x+n}} \]

\[ P_{x+n} = \frac{\sum_{i=0}^{n-1} d_{x+i} \cdot v^{i+1} + D \cdot K \cdot I_{x+i} \cdot v^{i}}{\sum_{i=0}^{n} I_{x+i} \cdot v^{i}} \]

\[ P_{x+n} = \frac{\sum_{i=0}^{n-1} q_x \cdot v^{i+1} + D \cdot K \cdot p_x \cdot v^{i}}{\sum_{i=0}^{n} p_x \cdot v^{i}} \]
Tutorial of MS Excel application

Input information:
Fill input information to yellow cells. See in part 1.

Output results:
Results are automatically calculated in green part 2.

Different ways of calculation:
See part 3, different approaches to obtain same result based on input information.

Detailed application of formulas:
Different approaches from part 3 are described in part 4 in detail.