Basic examples and Calculations in Life Insurance

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Example 1

A client, man 25 years old, wants to make an insurance contract for 25 years.

A. In case of death he wants to secure his family with 200 000. In case he survives all 25 years he also wants to receive 200 000. How much would this contract cost him?
Derive results of single and regular netto premium.
B. The client wants to see how much of the premium he pays for insurance to secure his family in case of death and how much for 200 000 in case he survives.
Derive results of single and regular premium.
C. You, as an insurance company, should be able to cover most of your contracts. For that purpose you should calculate reserves for all considered contracts. Calculate reserve in year 7.
D. Apply charges and calculate brutto premium and reserves of contracts mentioned in section A and B.

**note: assume traditional approach with interest rate 0.025 p.a.*

A. Endowment

Gender	Male
Age	25
Policy period	25
Death benefit (K)	200 000
Survival benefit (D)	200 000
Interest rate (i)	0.025
Type of insurance contract	Endowment
Premium:	Endowment
Single	108 888,99
Regular annual	5 829,87
Regular monthly	491,46

Single premium

Using Excel function

```
fx =Axn(25;25;0,025;200000;200000)*200000
```

Using Mortality tables

$$\Pi_{xn} = K * \frac{d_x * v + d_{x+1} * v^2 + \ldots + d_{x+n-1} * v^n}{l_x} + D * \frac{l_{x+n} * v^n}{l_x}$$

$$108\,888{,}99 = 200\,000 * \frac{2626{,}83}{98982{,}34} + 200\,000 * \frac{95039{,}86 * 0{,}53939}{98982{,}34}$$

Using probability

$$\Pi_{xn} = K * \left(q_x * v + {}_{1|}q_x * v^2 + \ldots + {}_{n-1|}q_x * v^n\right) + D * {}_np_x * v^n$$

$$108\,888{,}99 = 200\,000 * 0{,}02654 + 200\,000 * 0{,}96017 * 0{,}539391$$

Using Commutation numbers

$$\Pi_{xn} = K * \frac{C_x + C_{x+1} + C_{x+2} + \ldots + C_{x+n-1}}{D_x} + D * \frac{D_{x+n}}{D_x}$$

$$108\,888{,}99 = 200\,000 * \frac{1416{,}89}{53390{,}15} + 200\,000 * \frac{27651{,}11}{53390{,}15}$$

$$\Pi_{xn} = K * \frac{M_x - M_{x+n}}{D_x} + D * \frac{D_{x+n}}{D_x}$$

$$108\,888{,}99 = 200\,000 * \frac{1416{,}89}{53390{,}15} + 200\,000 * \frac{27651{,}11}{53390{,}15}$$

Regular premium – annual

Using Excel function

```
fx  =regular_Endowment(25;25;0,025;200000;200000)*200000
```

Using Actuarial formulas

$$K * {}_nP_x = \frac{K * A^1_{xn} + D * {}_nE_x}{\ddot{a}_{xn\rceil}}$$

$$5\,829{,}87 = \frac{200\,000 * 0{,}02654 + 200\,000 * 0{,}51791}{18{,}68}$$

Using probability

$$K * {}_nP_x = \frac{K * \left(q_x * v + {}_{1|}q_x * v^2 + \ldots + {}_{n-1|}q_x * v^n\right) + D * {}_np_x * v^n}{1 + {}_1p_x * v + \ldots + {}_{n-1}p_x * v^{n-1}}$$

$$5\,829{,}87 = \frac{200\,000 * 0{,}02654 + 200\,000 * 0{,}51791}{18{,}68}$$

Using Commutation numbers

$$K * {}_nP_x = \frac{K * (M_x - M_{x+n}) + D * D_{x+n}}{N_x - N_{x+n}}$$

$$5\,829{,}87 = \frac{200\,000 * 1416{,}89 + 200\,000 * 27651{,}11}{997208{,}20}$$

$$K * {}_nP_x = \frac{K * (d_x * v + d_{x+1} * v^2 + \ldots + d_{x+n-1} * v^n) + D * l_{x+n} * v^n}{l_x + l_{x+1} * v + l_{x+2} * v^2 + \ldots + l_{x+n-1} * v^{n-1}}$$

$$5\,829{,}87 = \frac{200\,000 * 2626{,}83 + 200\,000 * 51263{,}61}{1848768{,}26}$$

Regular premium – monthly

Frequency of premium (m)	monthly	m = 12

$$K * {}_nP_x^{(m)} = \frac{\text{Regular netto premium}}{m * \left(1 - \frac{(m-1) * (D_x - D_{x+n})}{2m * (N_x - N_{x+n})}\right)}$$

$$491{,}46 = \frac{5\,829{,}87}{11{,}86}$$

B. Pure endowment and Term insurance

Gender	Male		
Age	25		
Policy period	25		
Death benefit (K)	200 000		
Survival benefit (D)	200 000		
Interest rate (i)	0.025		
Type of insurance contract	Endowment / Term insurance		
Premium:	**Endowment**	**Pure endowment**	**Term insurance**
Single	108 888,99	103 581,31	5 307,67
Regular annual	5 828,87	5 545,70	284,17
Regular monthly	491,46	467,51	23,96

Pure endowment – single premium

Using Excel function

```
fx =nex(25;25;0,025)*200000
```

Using Mortality tables

$$\Pi_{xn} = D * \frac{l_{x+n} * v^n}{l_x}$$

$$103\,581{,}31 = 200\,000 * \frac{95039{,}86 * 0{,}53939}{98982{,}34}$$

Using probability

$$\Pi_{xn} = D * {}_np_x * v^n$$

$$103\,581{,}31 = 200\,000 * 0{,}96017 * 0{,}53939$$

Using Commutation numbers

$$\Pi_{xn} = D * \frac{D_{x+n}}{D_x}$$

$$103\,581{,}31 = 200\,000 * \frac{27651{,}11}{53390{,}15}$$

Pure endowment – Regular premium – annually

Using Excel function

```
fx  =regular_Pure_Endowment(25;25;0,025)*200000
```

Using Actuarial formulas

$$K * {}_{n}P_{x} = D * \frac{{}_{n}E_{x}}{\ddot{a}_{x\overline{n}|}}$$

$$5\,545,70 = 200\,000 * \frac{0,51791}{18,68}$$

Using Commutation numbers

$$K * {}_{n}P_{x} = D * \frac{D_{x+n}}{N_{x} - N_{x+n}}$$

$$5\,545,70 = 200\,000 * \frac{27651,11}{997208,20}$$

Using Mortality tables

$$K * {}_{n}P_{x} = D * \frac{l_{x+n} * v^{n}}{l_{x} + l_{x+1} * v + l_{x+2} * v^{2} + \ldots + l_{x+n-1} * v^{n-1}}$$

$$5\,545,70 = 200\,000 * \frac{51263,61}{1848768,26}$$

Using probabilities

$$K * {}_{n}P_{x} = D * \frac{{}_{n}p_{x} * v^{n}}{1 + {}_{1}p_{x} * v + \ldots + {}_{n-1}p_{x} * v^{n-1}}$$

$$5\,545,70 = 200\,000 * \frac{0,51791}{18,68}$$

Pure endowment – Regular premium –monthly

Frequency of premium (m)	monthly	m = 12

$$K * {}_nP_x^{(m)} = \frac{\text{Regular netto premium}}{m * \left(1 - \frac{(m-1) * (D_x - D_{x+n})}{2m * (N_x - N_{x+n})}\right)}$$

$$467{,}51 = \frac{5\ 545{,}70}{11{,}86}$$

Term insurance – Single premium

Using Excel function:

```
fx  =A1xn(25;25;0,025)*200000
```

Using Mortality tables

$$\Pi_{xn} = K * \frac{d_x * v + d_{x+1} * v^2 + \ldots d_{x+n-1} * v^n}{l_x}$$

$$5\ 307{,}67 = 200\ 000 * \frac{2626{,}83}{98982{,}34}$$

Using probabilities

$$\Pi_{xn} = K * q_x * v + {}_{1|}q_x * v^2 + \ldots + {}_{n-1|}q_x * v^n$$

$$5\ 307{,}67 = 200\ 000 * 0{,}02654$$

Using Commutation numbers

$$\Pi_{xn} = K * \frac{C_x + C_{x+1} + C_{x+2} \ldots + C_{x+n-1}}{D_x} = \frac{M_x - M_{x+n}}{D_x}$$

$$5\ 307{,}67 = 200\ 000 * \frac{1416{,}89}{53390{,}15} = \frac{1416{,}89}{53390{,}15}$$

Term insurance – Regular premium – annually

Using Excel function

```
fx =regular_Term_insurance(25;25;0,025)*200000
```

Using Actuarial formulas

$$ {}_nP_x = K * \frac{A1_{xn}}{\ddot{a}_{x\overline{n|}}} $$

$$ 284{,}17 = 200\ 000 * \frac{0{,}02654}{18{,}68} $$

Using Commutation numbers

$$ {}_nP_x = K * \frac{M_x - M_{x+n}}{N_x - N_{x+n}} = \frac{C_x + C_{x+1} + C_{x+2} \ldots + C_{x+n-1}}{N_x - N_{x+n}} $$

$$ 284{,}17 = 200\ 000 * \frac{1416{,}89}{997208{,}20} = \frac{1416{,}89}{997208{,}20} $$

Using Mortality tables

$$ {}_nP_x = K * \frac{d_x * v + d_{x+1} * v^2 + \ldots d_{x+n-1} * v^n}{l_x + l_{x+1} * v + l_{x+2} * v^2 + \ldots + l_{x+n-1} * v^{n-1}} $$

$$ 284{,}17 = 200\ 000 * \frac{2626{,}83}{1848768{,}26} $$

Using probabilities

$$ {}_nP_x = K * \frac{q_x * v + {}_{1|}q_x * v^2 + \ldots + {}_{n-1|}q_x * v^n}{1 + {}_1p_x * v + \ldots + {}_{n-1}p_x * v^{n-1}} $$

$$ 284{,}17 = 200\ 000 * \frac{0{,}02654}{18{,}68} $$

Term insurance – Regular premium - monthly

Frequency of premium (m)	Anually	m = 1

$$ K * {}_nP_x^{(m)} = \frac{\text{Regular netto premium}}{m * \left(1 - \frac{(m-1) * (D_x - D_{x+n})}{2m * (N_x - N_{x+n})}\right)} $$

$$ 23{,}96 = \frac{284{,}17}{11{,}86} $$

C. Reserves

Gender	Male
Age	25
Policy period	25
Death benefit (K)	200 000
Survival benefit (D)	200 000
Interest rate (i)	0.025
Reserve year (t)	7
Type of insurance contract	Endowment / Term insurance / Pure endowment
Premium:	**Reserve of regular premium**
Endowment	0,22055*200 000 =44 110
Pure endowment	0,21517*200 000 =43 034
Term insurance	0,00540*200 000 =1 080

Endowment

Using Excel function

```
fx =tVx_Endowment(25;25;7;0,025;1;200000;200000)*2000000
```

Using Actuarial formulas

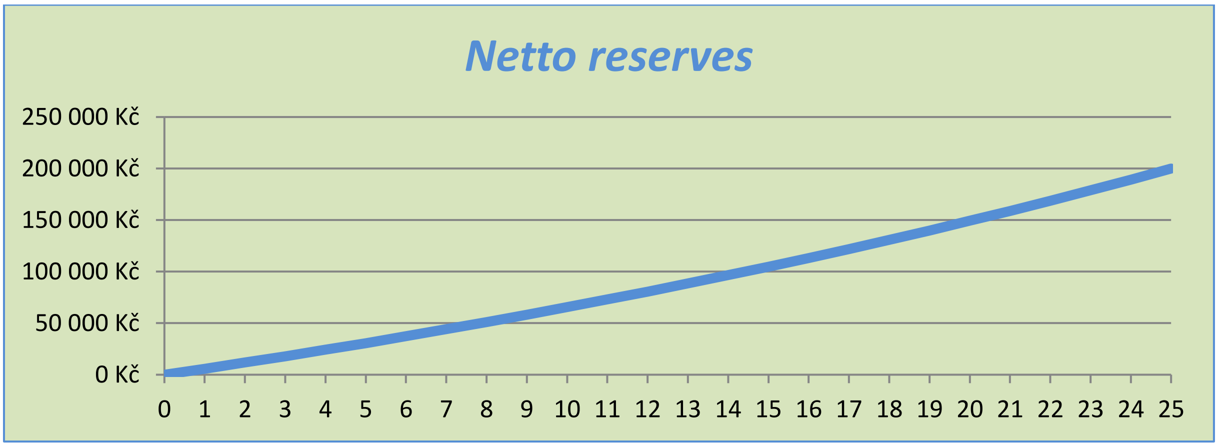
$${}_tV_x = A_{x+t,n-t\rceil} - {}_nP_x * \ddot{a}_{x+t,n-t\rceil}$$

$$0,22055 = 0,64492 - 0,02915 * 14,56$$

Using Commutation numbers

$${}_tV_x = 1 - \frac{D_x}{D_{x+t}} * \frac{N_{x+t} - N_{x+n}}{N_x - N_{x+n}}$$

$$0,22055 = 1 - \frac{53390,15}{44680,94} * \frac{650482,57}{997208,20}$$



Pure endowment

Using Excel function

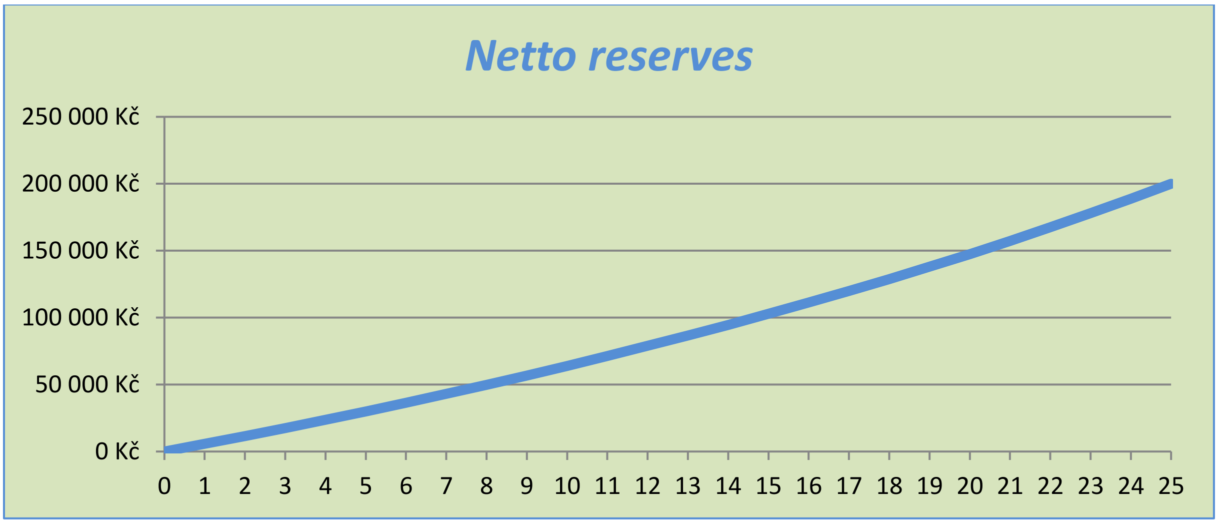
```
fx =tVx_Pure_endowment(25;25;7;0,025;1)*2000000
```

Using Actuarial formulas

$_tV_x$	=	$_{n-t}E_{x+t}$	-	$_nP_x$	*	$\ddot{a}_{x+t,n-t\rceil}$
0,21517	=	0,61886	-	0,02773	*	14,56

Using Commutation numbers

$_tV_x$	=	$\frac{D_{x+n}}{D_{x+t}}$	*	$\frac{N_x - N_{x+t}}{N_x - N_{x+n}}$
0,21517	=	$\frac{27651,11}{44680,94}$	*	$\frac{346725,63}{997208,20}$



Term insurance

Using Excel function:

`=tVx_Term_insurance(25;25;7;0,025;1)*2000000`

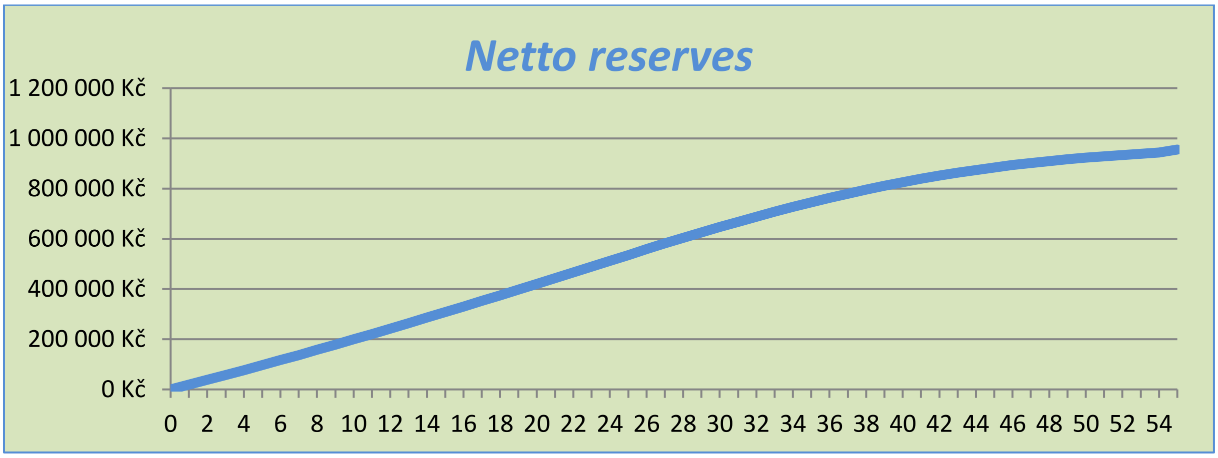
Using Actuarial formulas

$_tV_x$	=	$A^1_{x+t,n-t\rceil}$	-	$_nP_x$	*	$\ddot{a}_{x+t,n-t\rceil}$
0,0054	=	0,0261	-	0,0014	*	14,5584

Using Commutation numbers

$$_tV_x = \frac{M_{x+t} - M_{x+n}}{D_{x+t}} - \frac{M_x - M_{x+n}}{D_{x+t}} * \frac{N_{x+t} - N_{x+n}}{N_x - N_{x+n}}$$

$$0{,}0054 = \frac{1164{,}4039}{44680{,}9390} - \frac{1416{,}8875}{44680{,}9390} * \frac{650482{,}5696}{997208{,}2031}$$



D. Brutto premium and reserves

Gender	Male
Age	25
Policy period	25
Death benefit (K)	200 000
Survival benefit (D)	200 000
Interest rate (i)	0.025
Reserve year (t)	7
Type of insurance contract	Endowment / Term insurance / Pure endowment

Premium:	Regular brutto	Brutto reserve
Endowment	5 965,93	42 546,92
Pure endowment	5 680,87	41 471,95
Term insurance	402,92	-487,07

*Note: There are no Excel functions to calculate brutto premium

Endowment

Endowment brutto premium

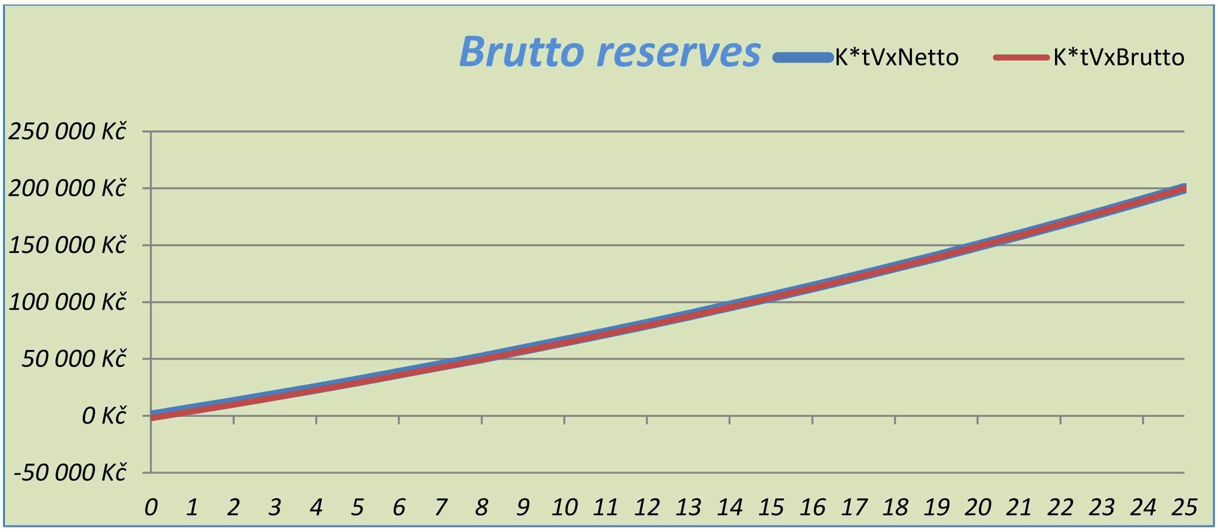
$$B_{xn} = \frac{K * A_{xn} + \alpha^{K} + \alpha^{fix} + \ddot{a}_{xn} * (\beta^{fix} + \beta^{K} + \gamma^{fix} + \gamma^{K})}{\ddot{a}_{xn} * (1 - \beta^{Bxn} - \gamma^{Bxn}) - \alpha^{Bxn}}$$

$$5\,965{,}93 = \frac{108888{,}99 + 2004{,}00 + 18{,}68 * 10{,}20}{18{,}68 * 0{,}99690 - 0{,}00020}$$

Endowment brutto reserve

$$K{*}V_x^{brutto} = K{*}{}_tV_x^{netto} - \frac{(\alpha^{K} + \alpha^{fix} + \alpha^{Bxn}) * \ddot{a}_{x+t,n-t}}{\ddot{a}_{x,n}}$$

$$42\,546{,}92 = 44109{,}87 - \frac{2005{,}19 * 14{,}56}{18{,}68}$$



Pure endowment

Pure endowment brutto premium

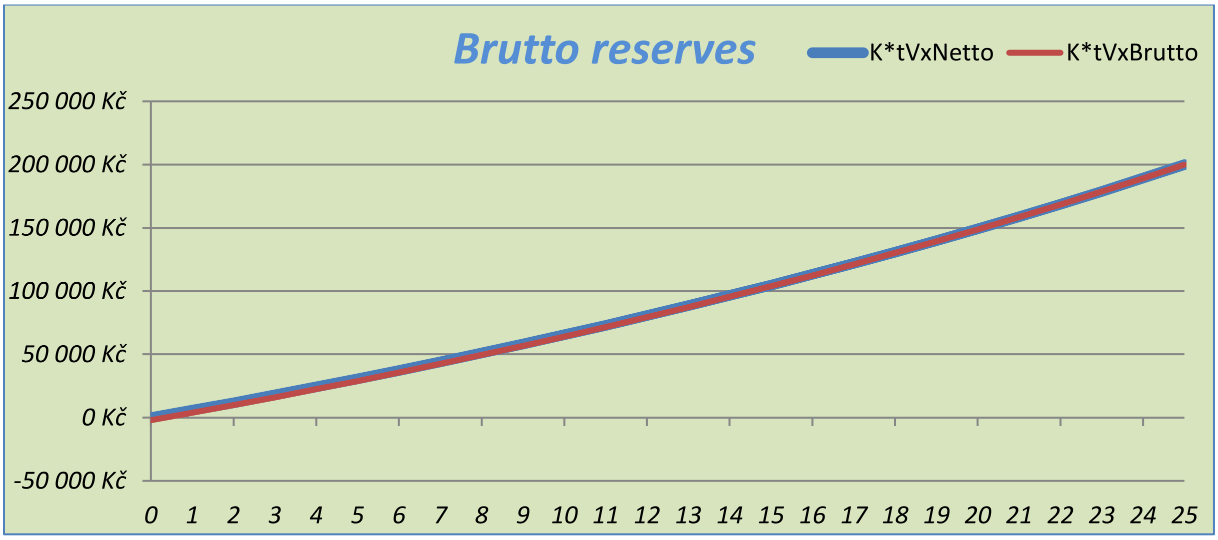
$$K * B_{xn}^{brutto} = \frac{D * {}_nE_x + \alpha^K + \alpha^{fix} + \ddot{a}_{xn} * (\beta^{fix} + \beta^K + \gamma^{fix} + \gamma^K)}{\ddot{a}_{xn} * (1 - \beta^{Bxn} - \gamma^{Bxn}) - \alpha^{Bxn}}$$

$$5\,680{,}87 = \frac{103581{,}31 + 2004{,}00 + 18{,}68 * 10{,}20}{18{,}68 * 0{,}99690 - 0{,}00020}$$

Pure endowment netto reserve

$$K*{}_tV_x^{brutto} = K*{}_tV_x^{netto} - \frac{(\alpha^K + \alpha^{fix} + \alpha^{Bxn}) * \ddot{a}_{x+t,n-t}}{\ddot{a}_{x,n}}$$

$$41\,471{,}95 = 43\,034{,}85 - \frac{2005{,}14 * 14{,}56}{18{,}68}$$



Term insurance

Term insurance brutto premium

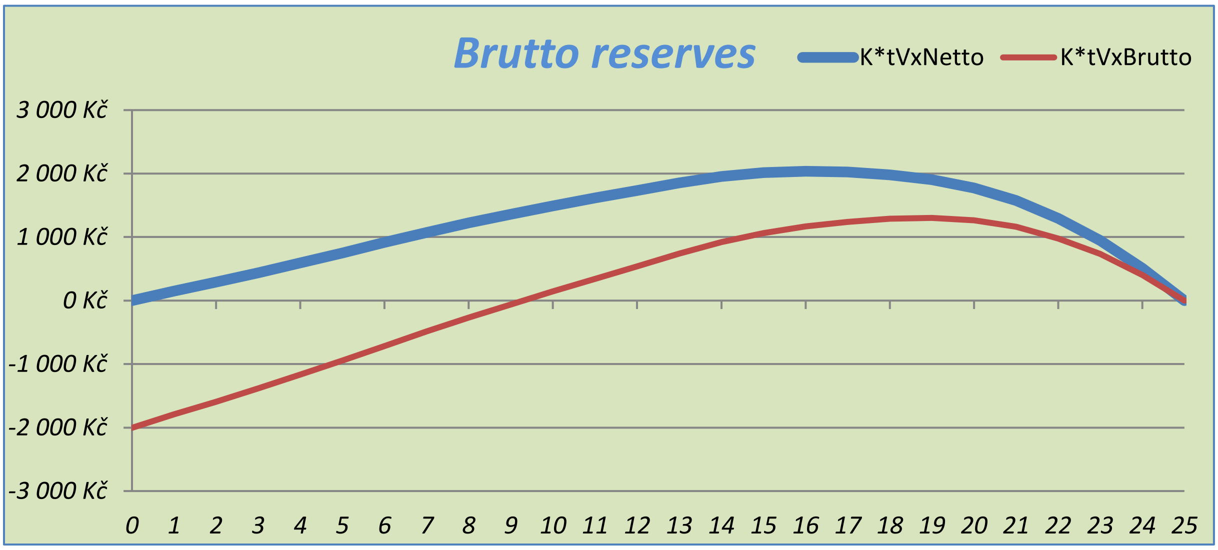
$$B_{xn} = \frac{K * A^1_{xn} + \alpha^K + \alpha^{fix} + ä_{xn} * (\beta^{fix} + \beta^K + \gamma^{fix} + \gamma^K)}{ä_{xn} * (1 - \beta^{Bxn} - \gamma^{Bxn}) - \alpha^{Bxn}}$$

$$402{,}92 = \frac{5307{,}67 + 2004{,}00 + 18{,}68 * 10{,}20}{18{,}68 * 0{,}99690 - 0{,}00020}$$

Term insurance brutto reserve

$${}_tV_x^{brutto} = {}_tV_x^{netto} - \frac{(\alpha^K + \alpha^{fix} + \alpha^{Bxn}) * \ddot{a}_{x+t,n-t}}{\ddot{a}_{x,n}}$$

$$-487{,}07 = 1075{,}01 - \frac{2004{,}08 * 14{,}56}{18{,}68}$$



Example 2

50 years old female wants to be insured for one million in case of death.

A. Compare single and regular premium for whole life.
B. Calculate also reserve at age 75.
C. Calculate whole life premium and reserve including charges.

A. Whole life

Gender	Female	
Age	50	
Policy period	ω	
Death benefit (K)	1 000 000	
Type of insurance contract	Whole life	
Premium:	**Single premium**	**Regular premium**
Whole life	453 687,44	20 254,98

Whole life – single premium

Using Excel function:

```
fx =A1x(50;0,025)*1000000
```

Using Mortality tables

$$\Pi_x = K * \frac{d_x * v + d_{x+1} * v^2 + \ldots}{l_x}$$

$$453\,687{,}44 = 1\,000\,000 * \frac{44291{,}15}{97624{,}80}$$

Using probability

$$\Pi_x = K * q_x * v + {}_{1|}q_x * v^2 + {}_{2|}q_x * v^3 + \ldots$$

$$453\,687{,}44 = 1\,000\,000 * 0{,}45369$$

Using commutation numbers

$$\Pi_x = K * \frac{C_x + C_{x+1} + C_{x+2} + \ldots}{D_x} = \frac{M_x}{D_x}$$

$$453\,687{,}44 = 1\,000\,000 * \frac{12886{,}16}{28403{,}17} = \frac{12886{,}16}{28403{,}17}$$

Whole life –Regular premium

Using Excel function:

`=regular_Whole_life(50;0,025)*1000000`

Using Actuarial formulas

$K * {}_nP_x$	=	K	*	$\dfrac{A_x}{\ddot{a}_{x\rceil}}$
20 254,98	=	1 000 000	*	$\dfrac{0,45369}{22,39881}$

Using Mortality tables

$K * {}_nP_x$	=	K	*	$\dfrac{d_x * v + d_{x+1} * v^2 + \ldots}{l_x + l_{x+1} * v + l_{x+2} * v^2 + \ldots + l_{x+n-1} * v^{n-1}}$
20 254,98	=	1 000 000	*	$\dfrac{44291,15}{2186679,82}$

Using probabilities

$K * {}_nP_x$	=	K	*	$\dfrac{q_x * v + {}_{1/}q_x * v^2 + \ldots}{1 + {}_1p_x * v + \ldots + {}_{n-1}p_x * v^{n-1}}$
20 254,98	=	1 000 000	*	$\dfrac{0,45}{22,40}$

Using Commutation numbers

$K * {}_nP_x$	=	K	*	$\dfrac{M_x}{N_x}$	=	$\dfrac{C_x + C_{x+1} + C_{x+2} + \ldots}{N_x}$
20 254,98	=	1 000 000	*	$\dfrac{12886,16}{636197,45}$	=	$\dfrac{12886,16}{636197,45}$

B. Reserves of Whole life

Gender	Female
Age	50
Policy period	ω
Death benefit (K)	1 000 000
Survival benefit (D)	0
Reserve (t)	25
Type of insurance contract	Whole life
Premium:	**Reserve**
Whole life	0,53529*1M = 535 287,41

Whole life – Reserve

Using Excel function:

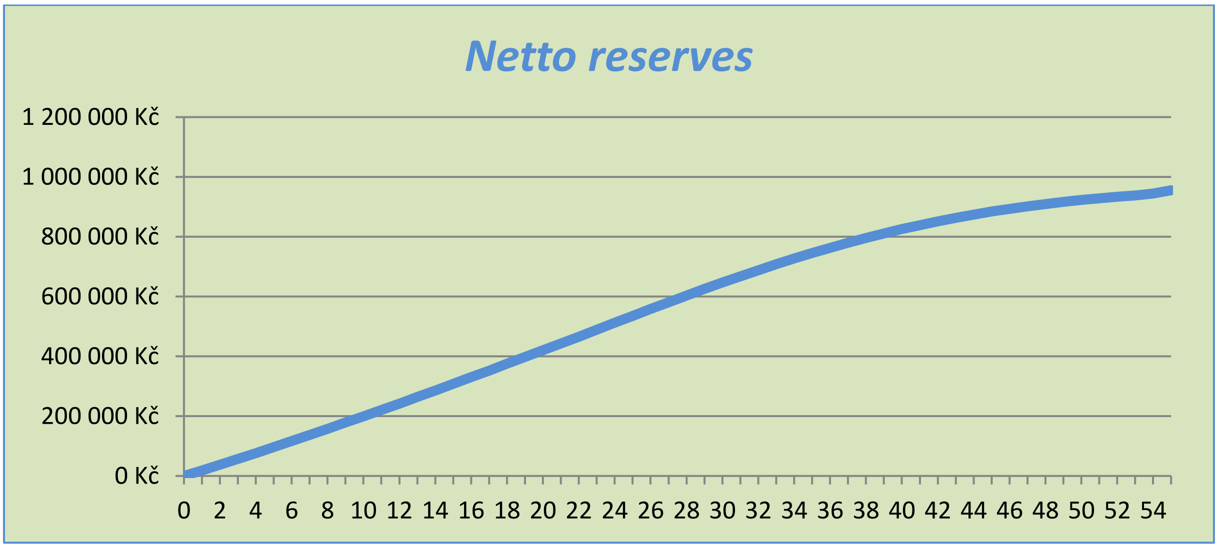
```
fx =tVx_Whole_live(50;25;0,025;1)*1000000
```

Using Actuarial formulas

${}_tV_x$	=	A^1_{x+t}	-	P_x	*	$\ddot{a}_{x+t}$
0,53529	=	0,74612	-	0,02025	*	10,40901

Using Commutation numbers

${}_tV_x$	=	1	-	$\frac{D_x}{D_{x+t}}$	*	$\frac{N_{x+t}}{N_x}$
0,53529	=	1	-	$\frac{28403,17}{12285,73}$	*	$\frac{127882,26}{636197,45}$



C. Brutto Whole life

Gender	Female	
Age	50	
Policy period	ω	
Death benefit (K)	1 000 000	
Reserve (t)	25	
Type of insurance contract	Whole life	
Premium:	**Regular brutto premium**	**Brutto reserve**
Whole life	20 443,72	534 346,79

Whole life – Regular brutto premium

Whole life brutto premium

$$B_{xn} = \frac{K * A^1_x + \alpha^K + \alpha^{fix} + ä_x * (\beta^{fix} + \beta^K + \gamma^{fix} + \gamma^K)}{ä_x * (1 - \beta^{Bxn} - \gamma^{Bxn}) - \alpha^{Bxn}}$$

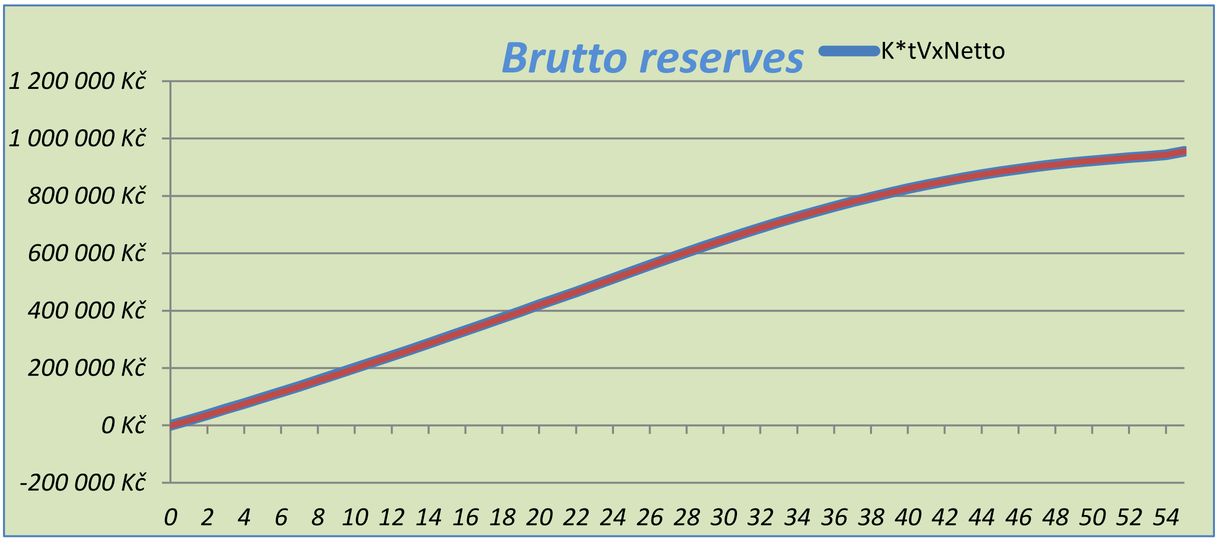
$$20\,443{,}72 = \frac{453687{,}44 + 2020{,}00 + 22{,}40 * 35{,}00}{22{,}40 * 0{,}99690 - 0{,}00020}$$

Whole life – Brutto reserve

Whole life brutto reserve

$$K * {}_tV_x^{brutto} = K * {}_tV_x^{netto} - \frac{(\alpha^K + \alpha^{fix} + \alpha^{Bxn}) * ä_{x+t}}{ä_x}$$

$$534\,346{,}79 = 535287{,}41 - \frac{2024{,}09 * 10{,}41}{22{,}40}$$



Example 3

Young man of age 30 wants to be secured when he reaches his 60 by certain amount of money.

A. How much he has to pay every year to get two million when he turns 60.
B. How much he would have to pay to get 5 000 every year from 60 until the rest of his life. Compare this premium with premium for 5 000 every year from his 60 until his 80.
C. He is sure that now he can pay max 2000 per year. What annuity he can expect when he turns 60 until his death and until his 80?
D. Make the reserves of these two contracts at year 12 and 45.

A .Pure endowment

Using Actuarial formulas

$$ {}_nP_x = D * \frac{{}_nE_x}{\ddot{a}_{x\overline{n}|}} $$

$$ 40\,680{,}47 = 2\,000\,000 * \frac{0{,}42621}{20{,}95} $$

B. Deferred annuity

Gender	Male	
Age (x)	30	
Policy period (N)	50/ω	
Deferred (k)	30	
Annuity (D)	5 000	
Interest rate (i)	0,025	
Type of insurance contract	Deferred / Whole life / Temporary annuity	
Premium:	**Single premium**	**Regular premium**
Whole life annuity	30 576,34	1 459,22
Temporary annuity	26 639,88	1 271,36

Whole life annuity – Single premium

Using Excel function

```
fx =dax(30;0,025;1;1;30)*5000
```

Using Mortality tables

$$ \Pi_x = D * \frac{l_{x+k+1} * v^{k+1} + l_{x+k+2} * v^{k+2} + \ldots}{l_x} $$

$$ 30\,576{,}34 = 5\,000 * \frac{603057{,}53}{98615{,}07} $$

Using probabilities

Π_x	=	D	*	${}_{k+1}p_x * v^{k+1} + {}_{k+2}p_x * v^{k+2} + \dots$
30 576,34	=	5 000	*	6,12

Using Commutation numbers

Π_x	=	D	*	$\frac{D_{x+k+1} + D_{x+k+2} \dots}{D_x}$	=	$\frac{N_{x+k+1}}{D_x}$
30 576,34	=	5 000	*	$\frac{2766364,06}{47014,01}$	=	$\frac{287503,27}{47014,01}$

Whole life – Regular premium

Using Excel function

```
fx  =regular_Whole_life_annuity(30;0,025;30;1)*5000
```

Regular premium

${}_nP_x$	=	$\frac{D * {}_{k+1\rvert}\ddot{a}_x}{\ddot{a}_{xk\rceil}}$
1 459,22	=	$\frac{30576,34}{20,95}$

Temporary annuity – Single premium

Using Excel function

```
fx  =Daxn(30;50;0,025;1;1;30)*5000
```

Using Mortality tables

Π_{xn}	=	D	*	$\frac{l_{x+k+1} * v^{k+1} + l_{x+k+2} * v^{k+2} + \dots + l_{x+n} * v^n}{l_x}$
26 639,88	=	5 000	*	$\frac{525418,84}{98615,07}$

Using probabilities

Π_{xn}	=	D	*	${}_{k+1}p_x * v^{k+1} + {}_{k+2}p_x * v^{k+2} + \ldots + {}_{n}p_x * v^{n}$
26 639,88	=	5 000	*	5,33

Using Commutation numbers

Π_{xn}	=	D	*	$\frac{D_{x+k+1} + D_{x+k+2} + \ldots + D_{x+n}}{D_x}$	=	$\frac{N_{x+k+1} - N_{x+n+1}}{D_x}$
26 639,88	=	5 000	*	$\frac{250489,59}{47014,01}$	=	$\frac{250489,59}{47014,01}$

Temporary annuity – Regular premium

Using Excel function

```
=regular_Temporary_annuity(30;50;0,025;30;1)*5000
```

Regular premium

${}_nP_x$	=	$\frac{D * {}_{k\|}a_{xn}}{\ddot{a}_{xk}}$
1 271,36	=	$\frac{26639,88}{20,95}$

C. Fixed premium annuity

Whole life

$$P \cdot \ddot{a}_{xk} = D \cdot {}_{k|}a_x$$

$$D = \frac{P \cdot \ddot{a}_{xk}}{{}_{k|}a_x}$$

$$D = \frac{2000 \cdot 20,95}{6,11} = 6852,95$$

Temporary annuity

$$P \cdot \ddot{a}_{xk} = D \cdot {}_{k|}a_{xn}$$

$$D = \frac{P \cdot \ddot{a}_{xk}}{{}_{k|}a_{xn}}$$

$$D = \frac{2000 \cdot 20{,}95}{5{,}32} = 7865{,}58$$

Excel function for $\ddot{a}_{xk}$

```
fx  =Daxn(30;30;0,025;0;1;0)
```

Excel function for ${}_{k|}a_{x}$

```
fx  =dax(30;0,025;1;1;30)
```

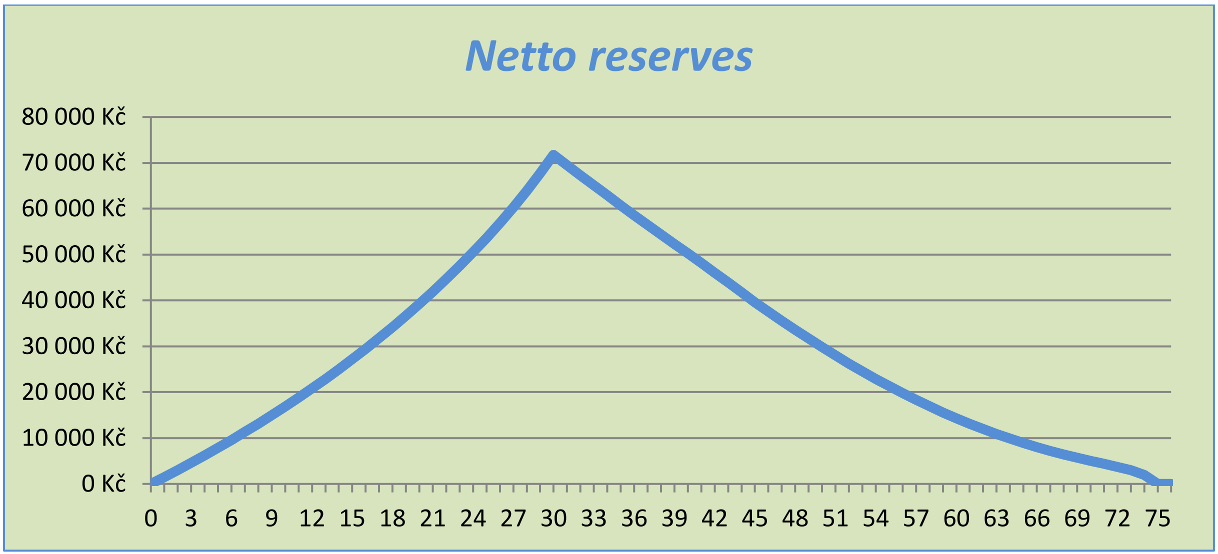
Excel function for ${}_{k|}a_{xn}$

```
fx  =Daxn(30;30;0,025;0;1;0)
```

D. Reserves

Gender	Male	
Age (x)	30	
Policy period (N)	50/ω	
Deferred (k)	30	
Annuity (D)	5 000	
Interest rate (i)	0,025	
Type of insurance contract	Deferred / Whole life / Temporary annuity	
Premium:	**Reserve t = 12**	**Reserve t = 45**
Whole life annuity	4,16*5 000 = 20 800	7,93*5 000 = 39 650
Temporary annuity	3,63* 5 000 = 18 150	3,98*5 000 = 19 900

Whole life annuity



Using Excel function:

```
=tVx_Whole_life_annuity(30;45;0,025;1;30)*5000
```

Using actuarial symbols:

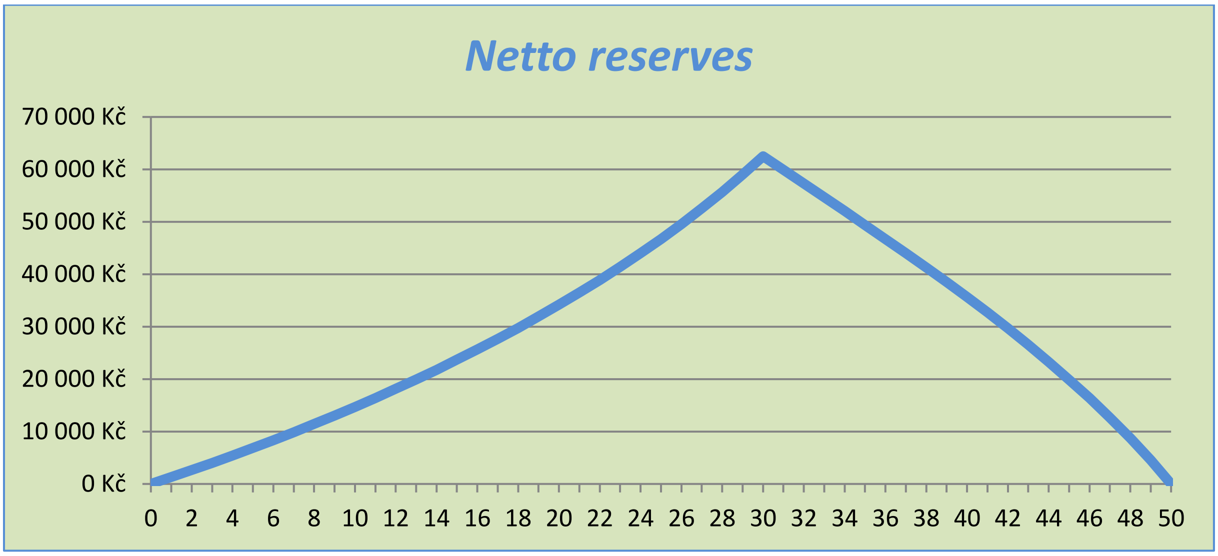
K=12, $_tV_x$ for k > t

$_tV_x$	=	$_{k-t\|}a_{x+t}$	-	$_kP_x$	*	$a_{x+t,k-t}$
4,16	=	8,34	-	0,29	*	14,30

K=45, $_tV_x$ $k \leq t$

$_tV_x$	=	a_{x+t}
7,93	=	7,93

Temporary annuity



Using Excel function:

```
fx =tVx_Temporary_annuity(30;50;12;0,025;1;30)*5000
```

Using actuarial symbols:

K=12, $_tV_x$ for $k > t$

$_tV_x$	=	$_{k-t\|}a_{x+t,n-t}$	-	$_{k\|n}P_x$	*	$a_{x+t,k-t}$
3,63	=	7,26	-	0,25	*	14,30

$K = 45$, ${}_tV_x$ for $k \leq t$

${}_tV_x$	=	$a_{x+t,n-t}$
3,98	=	3,98

Example 4 – Universal Traditional approach

30 years old man has very special demands about his insurance contract. When he turns 40, he wants to be insured for 10 000 in case of death. When he turns 50, he wants to increase death benefit up to 20 000 and until his 55 he wants to receive 5 000 every year. In his 60 he wants to get annuity 20 000 and then until his 70 to be insured for 10 000 in case of death but doesn't want to pay premium.

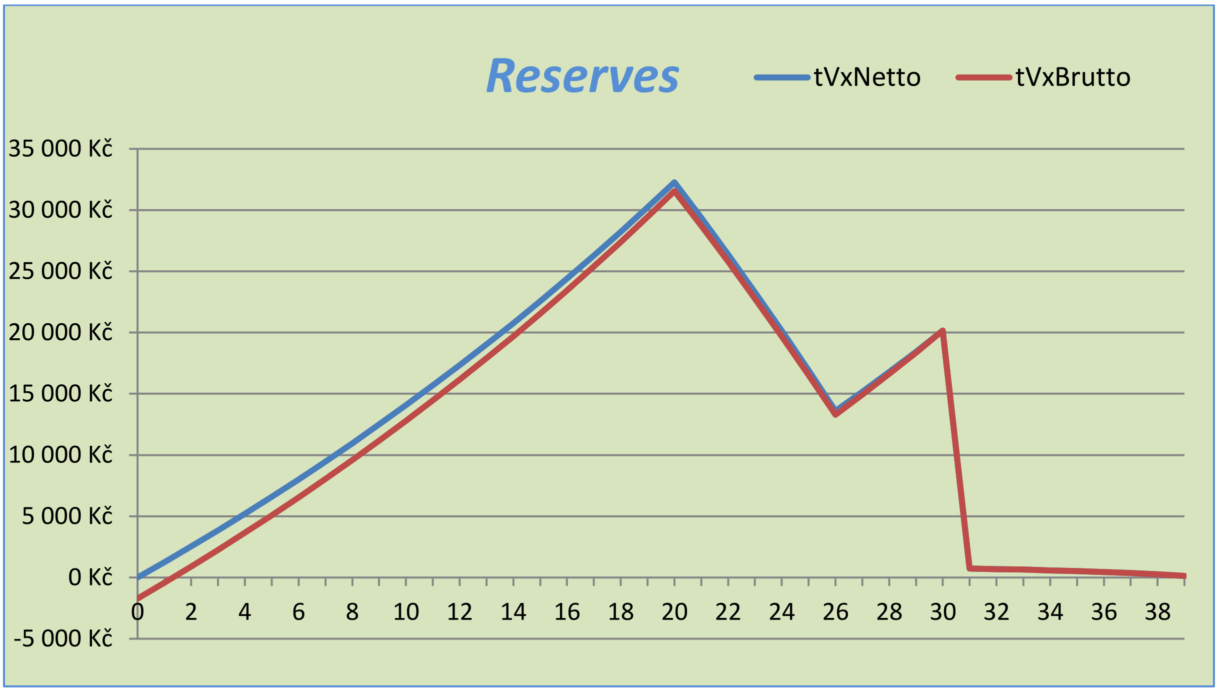
The charges for this contract assume following:

alfa fix	2000
alfa from premium (%)	2,0000%
alfa z K (%)	2,0000%
alfa z D (%)	2,0000%

	fix	*from K or D*	*from P*
Beta	20,00	2,00000%	2,00000%
Gamma	30,00	3,00000%	3,00000%
Delta	35,00	3,50000%	

Calculate netto and brutto premium and reserve in policy year 5 and 25 of this contract.

Gender	Male	
Age (x)	30	
Policy period (N)	40	
Deferred (k)	10	
Reserve (t)	5 / 25	
Death benefit (K)	From age 40 to 50: 10 000	
	From age 50 to 60: 20 000	
	From age 60 to 70: 10 000 without paying premium	
Annuity (D)	From age 50 to 55: 5 000	
	In age 60: 20 000	
Interest rate (i)	0,025	
Type of insurance contract	Deferred / Term insurance / Temporary annuity	
Premium:	**Netto**	**Brutto**
Premium	1 222,46	2 035,55
Reserve year 5	6 593,45	5 079,80
Reserve year 25	16 921,97	16 538,53



The input information of this policy can be seen on picture below.

Time (t)	Age (x)	In force	Premium transfer	Death benefit	Survival benefit
1	30	1	1		
2	31	1	1		
3	32	1	1		
4	33	1	1		
5	34	1	1		
6	35	1	1		
7	36	1	1		
8	37	1	1		
9	38	1	1		
10	39	1	1		
11	40	1	1	10 000	
12	41	1	1	10 000	
13	42	1	1	10 000	
14	43	1	1	10 000	
15	44	1	1	10 000	
16	45	1	1	10 000	
17	46	1	1	10 000	
18	47	1	1	10 000	
19	48	1	1	10 000	
20	49	1	1	10 000	
21	50	1	1	20 000	5 000
22	51	1	1	20 000	5 000
23	52	1	1	20 000	5 000
24	53	1	1	20 000	5 000
25	54	1	1	20 000	5 000
26	55	1	1	20 000	5 000
27	56	1	1	20 000	
28	57	1	1	20 000	
29	58	1	1	20 000	
30	59	1	1	20 000	
31	60	1		10 000	20 000
32	61	1		10 000	
33	62	1		10 000	
34	63	1		10 000	
35	64	1		10 000	
36	65	1		10 000	
37	66	1		10 000	
38	67	1		10 000	
39	68	1		10 000	
40	69	1		10 000	
41	70	0		10 000	
42	71	0			
43	72	0			

Example 5 – Flexible product

Deal flexible contract for 35 years old male for 30 years.

A. What is the capital value at the end of policy if the premium is 2 000 per year, death benefit is 100 000 and initial deposit of 1 000?
B. What is the minimal premium to keep zero reserve at the end of policy?
C. What is the premium for similar contract to endowment where the death benefit is 100 000 and survival benefit is 50 000?

A. Capital value at the end of policy

Policy characteristics

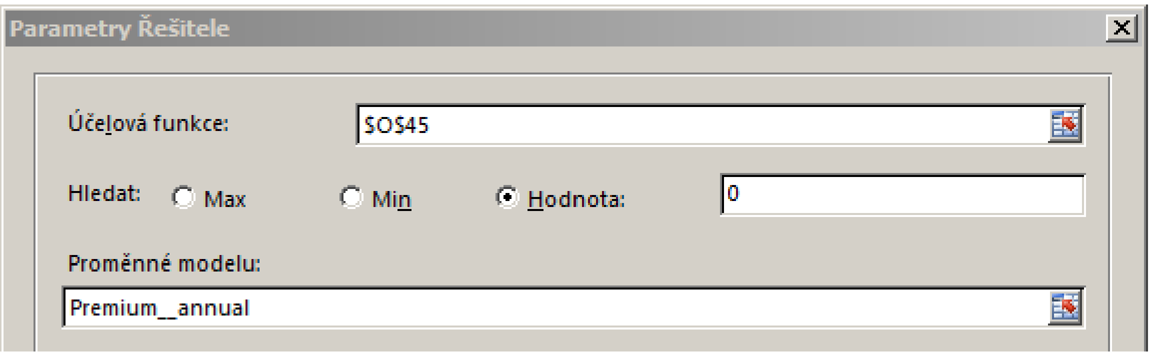
Claims:		
	Death	SA+CV
Tech. Int. rate		2,5%
Policy charges:		
	1st year	80,0%
	2+years	20,0%
Profit share		85,0%
Surrender fee		5,0%

Model point:

Year	2015
Age at valuation date	35
Sex	Male
SA	100 000
Premium (annual)	2 000
CV at val. date	1 000
Policy year at val. date	1
Policy_period	30

Year	Age	Policy Year	Pshare	CV EoY
...	...	...	...	...
2044	64	30	...	67 821

B. Minimizing premium[1]



Policy characteristics

Claims:		
	Death	SA+CV
Tech. Int. rate		2,5%
Policy charges:		
	1st year	80,0%
	2+years	20,0%
Profit share		85,0%
Surrender fee		5,0%

Modelpoint:

Year	2015
Age at valuation date	35
Sex	Male
SA	100 000
Premium (annual)	**538**
CV at val. date	1000
Policy year at val. date	1
Policy_period	30

[1] *Note: This Solution requires to install solver.

C. Endowment

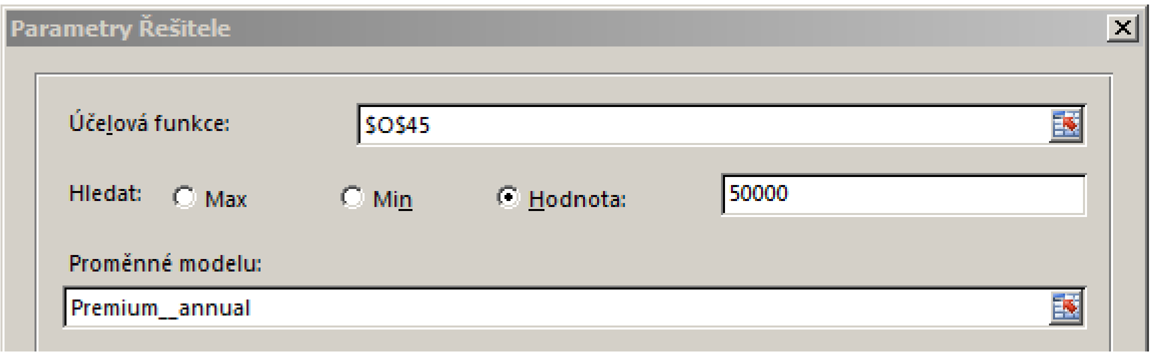
Policy charcteristics

Claims:		
	Death	SA
Tech. Int. rate		2,5%
Policy charges:		
	1st year	80,0%
	2+years	20,0%
Profit share		85,0%
Surrender fee		5,0%

Modelpoint:

Year	2015
Age at valuation date	35
Sex	Male
SA	100 000
Premium (annual)	***1 616***
CV at val. date	1 000
Policy year at val. date	1
Policy_period	30

Year	*Age*	*Policy Year*	*Pshare*	*CV EoY*
...	...	...	...	...
2044	**64**	**30**	...	**50 000**



Example 6 – Cash-Flow model

One big company wants to insure 1 000 its employees. In case of death, 100 000 will be paid and if insured person survives 10 years the contract will be canceled and the person will obtain at least 20 000. All employees are all men in age 35.

A. Compare traditional and flexible approach and decide which of the two types of contract is more profitable.
B. Make calculation of Liability Adequacy Test (LAT) for both approaches.

A. Profitability test

Flexible approach

Calculation of flexible product premium with SA = 100 000 and at the end CV = 20 000. By using solver the minimal premium is **2 495**.

Policy characteristics

Claims:	
Death	SA
Tech. Int. Rate	2,5%
Policy charges:	
1st year	80,0%
2+years	20,0%
Profit share	85,0%
Surrender fee	5,0%

Modelpoint:

Year	2015
Age at valuation date	35
Sex	M
SA	100 000
Premium (annual)	**2 495**
CV at val. date	0
Policy year at val. date	1
Policy_period	10

From Cash-Flow model the profitable criteria is Pcrit 1.

$$Pcrit1 = \frac{PVPL}{PV\,\mathrm{Premium}}$$

Liability model

PV CF	**-29 388 850**
PV PL	**-29 636 250**
PV Premium	**115 266 345**
Net PL	**-24 830 982**
Total Earnings	**-24 830 982**
Pcrit 1	**-25,7%**
Pcrit 2	**-118,8%**

Traditional approach

The premium of traditional approach of endowment contract below needs to be firstly calculated.

Time (t)	*Age (x)*	*In force*	*Premium transfer*	*Death benefit*	*Survival benefit*
1	35	1	1	100 000	
2	36	1	1	100 000	
3	37	1	1	100 000	
4	38	1	1	100 000	
5	39	1	1	100 000	
6	40	1	1	100 000	
7	41	1	1	100 000	
8	42	1	1	100 000	
9	43	1	1	100 000	
10	44	1	1	100 000	20 000

Regular brutto premium	2 146,58

The profit criteria can be seen in output of Cash-flow model.

Liability model

PV CF	**-14 196 104**
PV PL	**5 468 347**
PV Premium	**99 171 010**
Net PL	**4 106 974**
Total Earnings	**4 907 146**
Pcrit 1	**5,5%**
Pcrit 2	**25,5%**

Traditional approach seems to be more profitable for insurance company based on Pric1 and also for insured person because of lower premium. The reason why traditional approach gives better results is based on assumptions of no surrenders payoffs. If client cancels flexible policy, he receives his capital value adjusted by surrender fee. In traditional approach we assume no payout when the policy is canceled.

B. Liability Adequacy Test

Flexible approach

BE	29 388 850
RM	8 976 977
FV	38 365 827
Calculate LAT	
LAT	38 365 827

Traditional approach

BE	14 196 104
RM	10 882 614
FV	25 078 718
Calculate LAT	
LAT	5 074 425

After pressing “Calculate LAT” Liability adequacy test will be automatically calculated.

Actuarial formulas and MS Excel functions

Nomenclature

X	*Age*
N	*Policy period*
i	*Interest rate*
D	*Death benefit*
K	*Survival benefit*
t	*Time of reserve*
k	*Deferred time*

Mortality tables and Commutation tables

Probability of death q_x

Excel function: *qx (x)*

$$q_x = \frac{l_x - l_{x+1}}{l_x} = \frac{d_x}{l_x}$$

Probability of survive p_x

Excel function: *px (x)*

$$p_x = 1 - q_x$$

$$p_x = \frac{l_{x+1}}{l_x}$$

Number of living l_x

Excel function: *lx (x)*

$$l_x = p_{x-1} \cdot l_{x-1} = (1 - q_{x-1}) \cdot l_{x-1}$$

Number of death d_x

Excel function: *dx (x)*

$$d_x = l_x - l_{x+1}$$

Probability of surviving n years ${}_n p_x$

Excel function: *npx (x,n)*

$${}_n p_x = \frac{l_{x+n}}{l_x}$$

$$_n p_x = \prod_{i=0}^{n} p_{x+i}$$

Probability of death in n years $_nq_x$

Excel function: *nqx (x,n)*

$$_n q_x = \frac{l_x - l_{x+n}}{l_x}$$

$$_n q_x = 1 - {_n p_x}$$

Probability of death in certain age x+n $_{n|}q_x$

Excel function: n_qx (x,n)

$$_{n|} q_x = \frac{d_{x+n}}{l_x}$$

Discounted number of living at age x

Excel function: DDx (x,i)

$$D_x = l_x \cdot v^x$$

Discounted number of death at age x

Excel function: Cx (x,i)

$$C_x = d_x \cdot v^{x+1}$$

Commutation numbers of first order

Excel function: Mx (x,i)

$$N_x = D_x^{[2]} = \sum_{i=0}^{\omega-x} D_{x+i}$$

Excel function: Nx (x,i)

$$M_x = C_x^{[2]} = \sum_{i=0}^{\omega-x} C_{x+i}$$

Commutation numbers of second order

Excel function: Sx (x,i)

$$S_x = D_x^{[3]} = \sum_{i=0}^{\omega-x} N_{x+i}$$

Excel function: Rx (x,i)

$$R_x = C_x^{[3]} = \sum_{i=0}^{\omega-x} M_{x+i}$$

Actuarial functions

Pure endowment

Excel function: *nEx (x,n,i)*

$$ {}_nE_x = \frac{l_{x+n} \cdot v^n}{l_x} $$

$$ {}_nE_x = \frac{D_{x+n}}{D_x} $$

$$ {}_nE_x = {}_np_x \cdot v^n $$

Whole life

Excel function: *A1x (x,i)*

$$ A^1{}_x = \frac{\sum_{i=0}^{\omega} d_{x+i} \cdot v^{i+1}}{l_x} $$

$$ A^1{}_x = \frac{\sum_{i=0}^{\omega} C_{x+i}}{D_x} = \frac{M_x}{D_x} $$

$$ A^1{}_x = \sum_{i=0}^{\omega} q_{x+i} \cdot v^{i+1} $$

Temp insurance

Excel function: *A1xn (x,n,i)*

$$ A^1{}_{xn} = \frac{\sum_{i=0}^{n-1} d_{x+i} \cdot v^{i+1}}{l_x} $$

$$ A^1{}_{xn} = \frac{\sum_{i=0}^{n-1} C_{x+i}}{D_x} = \frac{M_x - M_{x+n}}{D_x} $$

$$ A^1{}_{xn} = \sum_{i=0}^{n-1} {}_{i|}q_{x+i} \cdot v^{i+1} $$

Endowment

Excel function: *Axn (x,n,I,D,K)*

$$ A_{xn} = A^1{}_{xn} + D/K \cdot {}_nE_x $$

$$ A_{xn} = \frac{\sum_{i=0}^{n-1} d_{x+i} \cdot v^{i+1}}{l_x} + D/K \cdot \frac{l_{x+n} \cdot v^n}{l_x} $$

$$A_{xn} = \frac{\sum_{i=0}^{n-1} C_{x+i}}{D_x} + D/K \cdot \frac{D_{x+n}}{D_x} = \frac{M_x - M_{x+n}}{D_x} + D/K \cdot \frac{D_{x+n}}{D_x}$$

$$A_{xn} = \sum_{i=0}^{n-1} {}_{i|}q_{x+i} \cdot v^{i+1} + D/K \cdot {}_n p_x \cdot v^n$$

Whole life annuity

Excel function: *Dax (x,i,in_arearrs,frequency,deferred)*

$${}_{k|}a_x = \frac{\sum_{i=1}^{\omega} l_{x+k+i} \cdot v^{k+i}}{l_x}$$

$${}_{k|}a_x = \sum_{i=1}^{\omega} {}_{k+i}p_x \cdot v^{k+i}$$

$${}_{k|}a_x = \frac{\sum_{i=1}^{\omega} D_{x+k+i}}{D_x} = \frac{N_{x+k+1}}{D_x}$$

$$a_x = \ddot{a}_x - 1$$

$${}_{k|}\ddot{a}_x = \frac{\sum_{i=0}^{\omega} l_{x+k+i} \cdot v^{k+i}}{l_x}$$

$${}_{k|}\ddot{a}_x = \sum_{i=0}^{\omega} {}_{k+i}p_x \cdot v^{k+i}$$

$${}_{k|}\ddot{a}_x = \frac{\sum_{i=0}^{\omega} D_{x+k+i}}{D_x} = \frac{N_{x+k}}{D_x}$$

Temp annuity

Excel function: *Daxn (x,N,i,in_arearrs,frequency,deferred)*

$${}_{k|}a_{xn} = \frac{\sum_{i=1}^{n} l_{x+k+i} \cdot v^{k+i}}{l_x}$$

$${}_{k|}a_{xn} = \sum_{i=1}^{n} {}_{k+i}p_x \cdot v^{k+i}$$

$${}_{k|}a_{xn} = \frac{\sum_{i=1}^{n} D_{x+k+i}}{D_x} = \frac{N_{x+k+n+1}}{D_x}$$

$$a_{xn} = \ddot{a}_{xn} - 1$$

$${}_{k|}\ddot{a}_{xn} = {}_{k-1|}a_{xn}$$

$$ {}_{k|}\ddot{a}_{xn} = \frac{\sum_{i=0}^{n-1} l_{x+k+i} \cdot v^{k+i}}{l_x} $$

$$ {}_{k|}\ddot{a}_{xn} = \sum_{i=0}^{n-1} {}_{k+i}p_x \cdot v^{k+i} $$

$$ {}_{k|}\ddot{a}_{xn} = \frac{\sum_{i=0}^{n-1} D_{x+k+i}}{D_x} = \frac{N_{x+k+n}}{D_x} $$

Reserves

Endowment reserve

Excel function: *tVx_ Endowment (x,n,t)*

$$ {}_tV_x = A_{x+t,n-t} - P_{zn} \cdot \ddot{a}_{x+t,n-t} $$

$$ {}_tV_x = 1 - \frac{D_x}{D_{x+t}} \cdot \frac{N_{x+t} - N_{x+n}}{N_x - N_{x+n}} $$

Whole life reserve

Excel function: *tVx_ Whole_live (x,t)*

$$ {}_tV_x = A^1{}_{x+t} - P_z \cdot \ddot{a}_{x+t} $$

$$ {}_tV_x = 1 - \frac{D_x}{D_{x+t}} \cdot \frac{N_{x+t}}{N_x} $$

Temp insurance reserve

Excel function: *tVx_ Temp_insurance (x,n,t)*

$$ {}_tV_x = A^1{}_{x+t,n-t} - P_{zn} \cdot \ddot{a}_{x+t,n-t} $$

$$ {}_tV_x = \frac{M_{x+t} - M_{x+n}}{D_{x+t}} - \frac{M_x - M_{x+n}}{D_{x+t}} \cdot \frac{N_{x+t} - N_{x+n}}{N_x - N_{x+n}} $$

Pure endowment reserves

Excel function: *tVx_ Pure_endowment (x,n,t)*

$$ {}_tV_x = {}_{n-t}E_{x+t} - P_{zn} \cdot \ddot{a}_{x+t,n-t} $$

$$ {}_tV_x = \frac{D_{x+n}}{D_{x+t}} \cdot \frac{N_x - N_{x+t}}{N_x - N_{x+n}} $$

Deferred life anuity reserves

Excel function:

For t<k

$$ {}_tV_x = {}_{k-t|}\ddot{a}_{x+t} - {}_kP_x \ddot{a}_{x+t,k-t} $$

$$ {}_tV_x = \frac{N_{x+k}}{D_{x+t}} \frac{N_x - N_{x+t}}{N_x - N_{x+k}} $$

For t>=k

$$ {}_tV_x = \ddot{a}_{x+t} $$

$$ {}_tV_x = \frac{N_{x+t}}{D_{x+t}} $$

Regular netto premium

Pure endowment regular

Excel function: *regular_Pure_endowment (x,n,i)*

$$ P_{xn} = \frac{{}_nE_x}{\ddot{a}_{xn}} $$

$$ P_{xn} = \frac{D_{x+n}}{N_x - N_{x+n}} $$

$$ P_{xn} = \frac{l_{x+n} \cdot v^n}{\sum_{i=0}^{n-1} l_{x+i} \cdot v^i} $$

$$ P_{xn} = \frac{{}_np_x \cdot v^n}{\sum_{i=0}^{n-1} {}_ip_x \cdot v^i} $$

Whole life regular

Excel function: *regular_Whole_life (x,n,i)*

$$ P_x = \frac{A_x}{\ddot{a}_x} $$

$$ P_x = \frac{M_x}{N_x} = \frac{\sum_{i=0}^{\omega} C_{x+i}}{N_x} $$

$$ P_x = \frac{\sum_{i=0}^{\omega} d_{x+i} \cdot v^{i+1}}{\sum_{i=0}^{\omega} l_{x+i} \cdot v^i} $$

$$P_x = \frac{\sum_{i=0}^{\omega} {}_{i|}q_x \cdot v^{i+1}}{\sum_{i=0}^{\omega} {}_i p_x \cdot v^i}$$

Temp insurance regular

Excel function: *regular_netto_Temp_insurance (x,n,i)*

$$P_{xn} = \frac{A^1{}_{xn}}{\ddot{a}_{xn}}$$

$$P_{xn} = \frac{M - M_{x+n}}{N_x - N_{x+n}} = \frac{\sum_{i=0}^{n-1} C_{x+i}}{N_x - N_{x+n}}$$

$$P_{xn} = \frac{\sum_{i=0}^{n-1} d_{x+i} \cdot v^{i+1}}{\sum_{i=0}^{n-1} l_{x+i} \cdot v^i}$$

$$P_x = \frac{\sum_{i=0}^{n-1} {}_{i|}q_x \cdot v^{i+1}}{\sum_{i=0}^{n-1} {}_i p_x \cdot v^i}$$

Endowment regular

Excel function: *regular_netto_Endowment (x,n,i,K,D)*

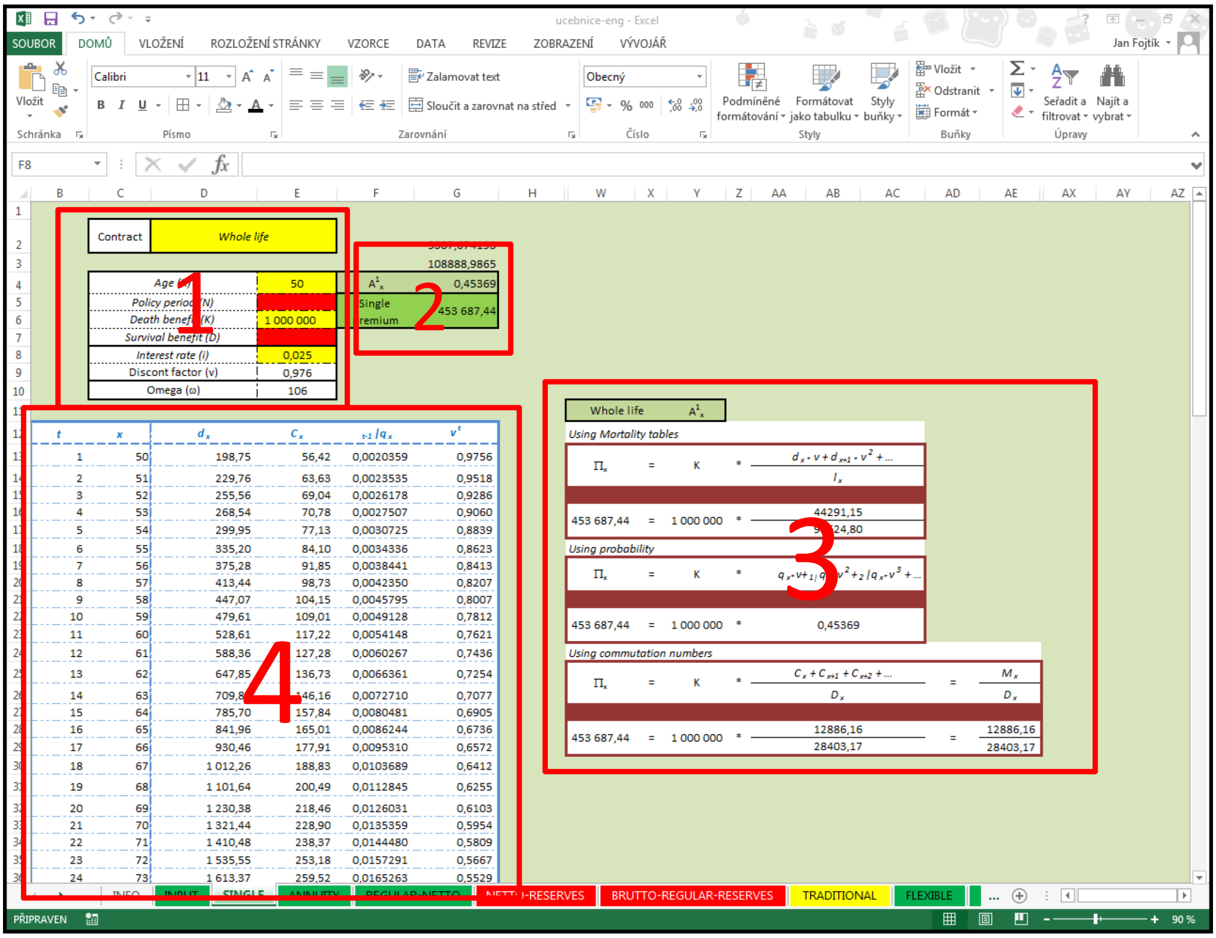
$$P_{xn} = \frac{A_{xn}}{\ddot{a}_{xn}} = \frac{A^1{}_{xn} + D/K \cdot {}_n E_x}{\ddot{a}_{xn}}$$

$$P_{xn} = \frac{M_x - M_{x+n} + D/K \cdot D_{x+n}}{N_x - N_{x+n}}$$

$$P_{xn} = \frac{\sum_{i=0}^{n-1} d_{x+i} \cdot v^{i+1} + D/K \cdot l_{x+n} \cdot v^n}{\sum_{i=0}^{n-1} l_{x+i} \cdot v^i}$$

$$P_{xn} = \frac{\sum_{i=0}^{n-1} {}_{i|}q_x \cdot v^{i+1} + D/K \cdot {}_n p_x \cdot v^n}{\sum_{i=0}^{n-1} {}_i p_x \cdot v^i}$$

Tutorial of MS Excel application



Input information:

Fill input information to yellow cells. See in part 1.

Output results:

Results are automatically calculated in green part 2.

Different ways of calculation:

See part 3, different approaches to obtain same result based on input information.

Detailed application of formulas:

Different approaches from part 3 are described in part 4 in detail.